Algebraic Geometry 1

Exercises Tutorium 5

Dr. Tom Bachmann Winter Semester 2020–21

Exercise 1. Let X be a topological space and S a set. Show that the presheaf

$$C(-, S) : U \mapsto C(U, S),$$

where $C(U, S)$ is the set of continuous maps from U to S, is a sheaf.

Exercise 2. Let X be a topological space and S a set. Consider the presheaf given by $c_S(U) = S$ for any U. Show that the *associated sheaf* assigns to $U \subset X$ the set of locally constant continuous maps from U to X.

Recall the notions of mono- and epimorphism in a category.

Exercise 3. Let $\alpha : F \to G$ be a morphism of presheaves on a topological space X. Show that α is an epimorphism (respectively monomorphism) if and only if $\alpha(U) : F(U) \to G(U)$ is, for every open subset U of X.

Hint: You may wish to recall representable presheaves and first show that a morphism of sets $F \to G$ is epi if and only if the two maps $G \to G \amalg_F G$ are equal.

Exercise 4. Let $\alpha : F \to G$ be a morphism of sheaves on a topological space X.

(1) Show that α is a monomorphism if and only if $\alpha(U) : F(U) \to G(U)$ is for every U.

(2) Show that α is an epimorphism if and only if the following holds: for every open U and $s \in G(U)$, there exists a covering $\{U_i\}$ of U and elements $t_i \in F(U_\alpha)$ such that $\alpha(t_i) = s|_{U_i}$.

Hint: You may use a formula for the associated sheaf of a presheaf.

Exercise 5. *Extra problem*: Give an example of a morphism of sheaves which is an epimorphism, but not an epimorphism when viewed as a morphism of presheaves.

Exercise 6. *Extra problem*: Give an example showing that the forgetful functor from sheaves to presheaves does not preserve coproducts (in general).