Algebraic Geometry 1

Exercises Tutorium 4

Dr. Tom Bachmann Winter Semester 2020–21

Throughout k is an algebraically closed field. Recall that $S = k[X_0, \ldots, X_n]$ and $S_+ = (X_0, \ldots, X_n)$.

Exercise 1. Let $I \subset S$ a homogeneous ideal, $f \in S$ a non-constant homogeneous polynomial with $f(P) = 0$ for all $P \in Z^h(I)$. Show that $f^q \in I$ for some q.

Exercise 2. Let $Z \subset \mathbb{P}^n$ be closed of dimension $n-1$. Show that $Z = Z^h(f)$ for some homogeneous $f \in S$.

Exercise 3. Let $I \subset S$ be a homogeneous radical ideal. Recall the embedding $\mathbb{A}^n \subset \mathbb{P}^n$. Show that $Z^h(I) \cap \mathbb{A}^n$ has coordinate ring the degree zero part of $S/I[x_0^{-1}]$.

Exercise 4. Let $S_+ \neq I \subset S$ be a homogeneous radical ideal. Show that $\dim Z^h(I) + 1 = \dim Z(I)$.