Algebra 2

Exercises Tutorium 9

Dr. Maksim Zhykhovich
Dr. Tom Bachmann

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Exercise 1. Let A be a Noetherian ring and let M be a finitely generated A-module. Show that every surjective homomorphism $\varphi : M \to M$ of A-modules is an isomorphism.
Hint: Show that submodules of M satisfy the ascending chain condition, and apply this to the sequence of modules $Ker(\varphi^{\circ n})$.

Exercise 2. Suppose given short exact sequences of abelian groups

$$0 \to \mathbb{Z} \to A_1 \to \mathbb{Z}/2 \to 0,$$
$$0 \to \mathbb{Z}/2 \to A_2 \to \mathbb{Z} \to 0,$$
$$0 \to \mathbb{Z}/2 \to A_3 \to \mathbb{Z}/2 \to 0.$$

Determine (up to isomorphism) possible groups A_i.

Exercise 3. Compute explicitly the following tensor products:
(1) $\mathbb{Z}/p \otimes_{\mathbb{Z}/pq} \mathbb{Z}/q$, where p and q are distinct primes.
(2) $\mathbb{Z}/n \otimes_{\mathbb{Z}} \mathbb{Q}/\mathbb{Z}$
(3) $\mathbb{Q} \otimes_{\mathbb{Z}} \mathbb{Q}$
(4) $\mathbb{Q} \otimes_{\mathbb{Z}} \mathbb{Q}/\mathbb{Z}$
(5) $\mathbb{Q}/\mathbb{Z} \otimes_{\mathbb{Z}} \mathbb{Q}/\mathbb{Z}$