Algebra 2

Exercises Tutorium 6

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Exercise 1. Let $n \geq 5$.

(1) Show that A_n has no proper subgroup of index $< n$.
Hint: Assume that such a subgroup H exists. Consider an action of A_n on the set A_n/H.

(2) Let K be a field. Let $P \in K[X]$ be a separable polynomial of degree n and let L be a splitting field of P over K. Show that if $\mathrm{Gal}(L/P) = A_n$ then every element of $L \backslash K$ has degree $\geq n$ over K.

Exercise 2.   (1) Let n be a positive integer. Determine the prime spectra of $\mathbb{Z}/n, \mathbb{Z}_{(n)}, \mathbb{Z}[1/n]$ as well as their maps to $Spec(\mathbb{Z})$.

(2) Determine the prime spectrum of $\mathbb{Z}[i]$ and its map to $Spec(\mathbb{Z})$.

Exercise 3. Let A be an abelian group and p a prime. Show that if $A[1/p] = 0$ and $A_{(p)} = 0$ then $A = 0$.