

Algebra 2

Exercises Tutorium 12

Dr. Maksim Zhykhovich
Dr. Tom Bachmann

Summer Semester 2020
13.07 - 17.07.2020

Exercise 1. Let $K \subset L$ be a finite Galois extension with Galois group G . Show that the map

$$\begin{aligned} L \otimes_K L &\longrightarrow \prod_{\sigma \in G} L \\ a \otimes b &\longmapsto (a \cdot \sigma(b))_{\sigma \in G} \end{aligned}$$

is an isomorphism of L -algebras, where the structure of an L -algebra on $L \otimes_K L$ is given by the multiplication on the first factor.

Hint: Use Exercise 1, Tutorium 10.

Exercise 2. Determine the integral closures of $k[x, y]/(y^2 - x^3)$ and $k[x, y]/(y^2 - x^3 - x^2)$.

If there is time left after questions:

Exercise 3. (1) Let K/k be a finite Galois extension with group G , and A a k -algebra. Show that $(A \otimes_k K)^G = A$.

(2) Describe the map $\text{Spec}(\mathbb{C}[X]) \rightarrow \text{Spec}(\mathbb{R}[X])$.