Algebra 2

Exercises Tutorium 10

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Exercise 1. Let $f : A \to B$ be a homomorphism of rings and I be an ideal in $A[X_1, ..., X_n]$. Denote by I_B the ideal generated by the image of I in $B[X_1, ..., X_n]$. Show that

$$B \otimes_A A[X_1, ..., X_n]/I \simeq B[X_1, ..., X_n]/I_B$$

as B-algebras.

Hint: One may start with the case $I = 0$.

Exercise 2. Compute explicitly the following tensor products:
(1) $\mathbb{Q} \otimes_{\mathbb{Z}} \mathbb{Q}$
(2) $\mathbb{C} \otimes_{\mathbb{R}} \mathbb{C}$
(3) $\mathbb{Q}[\sqrt[3]{2}] \otimes_{\mathbb{Q}} \mathbb{Q}[\sqrt[3]{2}]$
Hint: Use Exercise 1 for questions (2) and (3).

Exercise 3. Let A be a ring and I be a principal ideal in A. Show that A/I is a flat A-module if and only if I is generated by an idempotent element $e \in A$ (that is $e^2 = e$).
Hint: Use without proof that $A/I \otimes_A M \simeq M/IM$ (see Exercise 1 from this week's Exercise Sheet 9).