

Algebra 2

Tutorium 8

Prof. Markus Land
Dr. Maksim Zhykhovich

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Exercise 1. Let R be a commutative ring, $I \subset R$ a finitely generated ideal of R . Show that the following conditions are equivalent:

- (1) R/I is flat over R .
- (2) I is principal and generated by an idempotent.
- (3) R/I is projective over R .

Exercise 2. Show that \mathbb{Q} is not projective as \mathbb{Z} -module.

Exercise 3. (1) Let A be a local ring, M and N finitely generated A -modules. Show that if $M \otimes_A N = 0$, then $M = 0$ or $N = 0$.

Hint: Use Lemma 6.20 from the lecture.

(2) Assume A is not local. Find a counterexample to the statement from (1).