Algebra 2

Tutorium 5

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Exercise 1. Let A be a commutative ring and M an A-module. Let M_1, M_2 be submodules of M. Assume that M/M_1 and M/M_2 are Noetherian. Show that $M/(M_1 \cap M_2)$ is Noetherian.

Exercise 2. Let $n, m \in \mathbb{Z}$. Show that $\mathbb{Z}/n\mathbb{Z} \otimes_{\mathbb{Z}} \mathbb{Z}/m\mathbb{Z} \simeq \mathbb{Z}/d\mathbb{Z}$, where d is the greatest common divisor of n and m.

Exercise 3. Let A be a commutative ring, M, N two A-modules. Assume that N is free with A-basis $\{n_i\}_{i \in I}$.
a) Show that:

$$\sum_{i \in I} m_i \otimes n_i = 0 \text{ in } M \otimes_A N \iff m_i = 0 \text{ for all } i \in I\,,$$

where $m_i \in M$ and $m_i \neq 0$ only for finitely many $i \in I$.
b) Assume that M is also free with A-Basis $\{m_j\}_{j \in J}$.
Show that: $M \otimes_A N$ ist a free A-module with basis $\{m_j \otimes n_i \mid j \in J, i \in I\}$.