Algebra 2

Tutorium 1

Prof. Markus Land
Dr. Maksim Zhykhovich

Summer Semester 2023
27.04.2023

Exercise 1. Let $\mathfrak{a}$, $\mathfrak{b}$ and $\mathfrak{c}$ be ideals of a commutative ring A.
(1) Assume that $\mathfrak{a}$, $\mathfrak{b}$ are coprime and $\mathfrak{a}$, $\mathfrak{c}$ are also coprime. Show that $\mathfrak{a}$ and $\mathfrak{b}\mathfrak{c}$ are coprime.
(2) Show that if $\mathfrak{a}$ and $\mathfrak{b}$ are coprime, then $\mathfrak{a}^n$ and $\mathfrak{b}^m$ are also coprime for any positive integers n and m.

Exercise 2. Let A be a commutative ring with exactly one maximal ideal $\mathfrak{m} \subseteq A$. Let x be an element in A, such that $x^2 = x$. Show that $x \in \{0, 1\}$.

Exercise 3. Let R be a Boolean ring, that is a ring in which $x^2 = x$ for every $x \in R$. Show that
(1) $2x = 0$ for every $x \in R$.
(2) R is commutative.
(3) every prime ideal $I \subseteq R$ is maximal and $R/I \simeq \mathbb{Z}/2\mathbb{Z}$.
(4) Every finitely generated ideal in R is principal.