

## Excercise Sheet 7

Let  $A$  be a self-adjoint operator on a Hilbert space  $\mathcal{H}$ .

### Übung 1

Let  $U(t) = e^{itA}$ ,  $t \in \mathbb{R}$ . Show that  $A$  is bounded if and only if  $\lim_{h \rightarrow 0} \|U(h) - \text{id}_{\mathcal{H}}\| = 0$ .

### Übung 2

Suppose that  $A$  is non-negative and  $\text{Ker}A = 0$ . Let  $\psi_0 \in D(A)$ ,  $\psi_1 \in D(A^{1/2})$ . Prove that

$$\psi(t) := \cos(A^{1/2}t)\psi_0 + A^{-1/2} \sin(A^{1/2}t)\psi_1, \quad t \in \mathbb{R}.$$

is the unique solution of the Cauchy problem for the abstract wave equation

$$\begin{aligned} \psi''(t) &= -A\psi(t), \quad t \in \mathbb{R} \\ \psi(0) &= \psi_0, \quad \psi'(0) = \psi_1. \end{aligned}$$

Compute  $\psi'(t)$ ,  $t \in \mathbb{R}$ .