Mathematisches Institut der LMU
Dr. Heribert Zenk
D.T. Nguyen

Mathematische Quantenmechanik II
Sommer Semester 2018
17.5.2018

Excercise Sheet 6

Übung 1

Let $\mathcal{H}_1$, $\mathcal{H}_2$ be two Hilbert spaces. Let A be self–adjoint operator on $\mathcal{H}_1$ and B be symmetric operator and A–bounded with relative bound $\epsilon > 0$, i.e. $D(A) \subset D(B)$ and

$$\|B[\phi]\| \leq \epsilon \|A[\phi]\| + C_\epsilon \|\phi\| \quad \forall \phi \in D(A).$$

a) Prove that $A \otimes \mathrm{id}_{\mathcal{H}_2}$ with domain the span$\{\phi \otimes \psi : \phi \in D(A), \psi \in \mathcal{H}_2\}$ and $(A \otimes \mathrm{id}_{\mathcal{H}_2})[\phi \otimes \psi] = A\phi \otimes \psi$, is essentially self–adjoint on $\mathcal{H}_1 \otimes \mathcal{H}_2$.

b) Prove that the closure of $B \otimes \mathrm{id}_{\mathcal{H}_2}$ on $D(A)$ is relatively bounded with respect to $\overline{A \otimes \mathrm{id}_{\mathcal{H}_2}}$ with the relative bound ϵ.

Übung 2

Let A and B be operators on Hilbert spaces $\mathcal{H}_1$ and $\mathcal{H}_2$, respectively. Prove that

a) If $A \otimes B$ is different from zero, then $A \otimes B$ is symmetric if and only if there exists a $c \in \mathbb{K}$, $c \neq 0$ for which cA and $c^{-1}B$ are symmetric.

b) $A \otimes \mathrm{id}_{\mathcal{H}_2} + \mathrm{id}_{\mathcal{H}_1} \otimes B$ is symmetric if and only if there exists a $c \in \mathbb{R}$ for which $A - ic\mathrm{id}_{\mathcal{H}_1}$ and $B + ic\mathrm{id}_{\mathcal{H}_2}$ are symmetric.