

Embedding the free-choice semantics of AND/XOR-EPCs into the Boolean semantics

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Abstract. Each Event-driven Process Chain (EPC) translates into a free-choice system if its control flow branches and joins only at AND- or XOR-connectors. This free-choice system defines the free-choice semantics of the AND/XOR-EPC. But free-choice systems are not capable to deal with OR-connectors. Therefore a general EPC with OR-connectors obtains a semantics not until it has been translated into a certain coloured Petri net, named a Boolean system. This Boolean system defines the Boolean semantics of the EPC. We show that for well-behaved AND/XOR-EPCs the Boolean semantics reduces to the free choice semantics in as far as the Boolean system contains the free-choice system. To prove this result we introduce the concept of non-blocking components in live and safe free-choice systems. For each non-blocking component of the free-choice system we then construct a well-behaved bipolar system (bp-system), which is a particular Boolean system. We link the bp-systems of all non-blocking components of a covering to a coloured Petri which is named a linked bp-system. Its semantics is the Boolean semantics of the AND/XOR-EPC.

Keywords: Bipolar system, EPC, free-choice system, linked bp-system, non-blocking component.

1. Introduction

Free-choice systems form an important class of ordinary Petri nets. They are best analyzed and understood, and the theory of free-choice systems is both deep and elegant [DE1995]. Also for commercial applications of information management free-choice systems play an important role. In the context of Business Process Management (BPM) they serve to formalize process languages which have been introduced in a more informal way and lacked a well-defined semantics before.

The process modelling language most widespread in German commercial projects is the language of Event-driven Process Chains (EPC). It has been introduced by Keller, Nüttgens and Scheer in 1992 [KNS1992, Sch1994]. EPCs represent the control flow of a process as the interplay of three components: Events, functions and logical rules. The rules use connectors of logical type AND, XOR and OR. More specific, concurrency is represented by AND-splits and AND-joins. Strong or exclusive alternatives are modelled by XOR-splits and XOR-joins, while OR-splits and OR-joins model weak alternatives. All EPCs in this paper will be considered with a non-empty set of distinguished events, the initial events of the process.

The present paper deals mainly with AND/XOR-EPCs, i.e. with the restricted class of EPCs using only connectors of logical type AND or XOR. Each AND/XOR-EPC translates at once into a free-choice system FS : Functions and AND-connectors of

the EPC translate into transitions while events and XOR-connectors translate into places of FS . Each initial event of the EPC is marked by a token on the corresponding place of FS . The free-choice semantics of the AND/XOR-EPC is defined as the semantics of FS [Aal1999].

But the language of free-choice system is not capable to formalize EPCs with OR-connectors. Therefore we have introduced in a previous paper Boolean systems, a class of simple coloured Petri nets [LSW1998]. Boolean systems have two types of tokens, high tokens and low tokens. The low tokens serve to skip actions and to complete the marking of all pre-sets of a logical transition before a decision about its actual firing mode is possible. With the help of formulas from propositional logic the transitions of Boolean systems control the flow of the high tokens (*true*) and the low tokens (*false*).

A general EPC translates at once into a Boolean system BS : Functions and logical connectors of the EPC translate into transitions and events translate into places of BS . Each initial event of the EPC is marked by a high token at the corresponding place of BS and if necessary a suitable set of low tokens is added.

Those Boolean systems, which are needed for the restricted class of AND/XOR-EPCs, have been invented already in 1984 by Genrich and Thiagarajan [GT1984]. They named them Bipolar Synchronization Schemes, today abbreviated as bipolar systems (bp-system).

How do these two types of Petri nets, bipolar systems and free-choice systems, relate?

It turns out that each bp-system BS has a free-choice companion FS and a canonical morphism $high: BS \longrightarrow FS$ which maps the flow of high tokens of the coloured Petri net BS onto the flow of tokens of the free-choice system FS . Both systems are equivalent in as far as FS is well-behaved if and only if BS is well-behaved. In that case the morphism has the lifting property, i.e., it lifts occurrence sequences of FS to occurrence sequences of BS [Weh2010].

Yet, this equivalence holds only under the restriction that the behaviour of the free-choice system is fair. Here fairness is conceived as the absence of frozen tokens. That type of fairness is even a structural property, named non-blocking.

Therefore the present paper investigates a generalization of the above mentioned relation between the two classes of Petri nets. Relinquishing the non-blocking condition we prove:

For each well-behaved free-choice system FS a well-behaved linked bp-system LBS and a morphism

$$high: LBS \longrightarrow FS$$

exist, which maps the flow of high tokens of the coloured Petri net LBS onto the token flow of the free-choice system FS and satisfies the lifting property (Theor. 16 and Prop. 18).

As a consequence: For an AND/XOR-EPC, which translates into a well-behaved free-choice system FS , a well-behaved linked bp-system LBS exists

with $high(LBS) = FS$. We define the Boolean semantics of the EPC as the semantics of the coloured Petri net LBS . As a consequence, the free-choice semantics and the Boolean semantics of well-behaved AND/XOR-EPCs are equivalent. And this results allows us to consider the Boolean semantics of general EPCs a proper generalization of the free-choice semantics of AND/XOR-EPCs.

During our way in this paper we introduce two new concepts for Petri nets: Firstly *non-blocking components* of well-behaved free-choice systems and secondly the *linking* of bp-systems with respect to a family of morphisms.

2. Free-choice systems

For the convenience of the reader and to fix the notations we recall some fundamental concepts from the theory of ordinary Petri nets and define the subclass of free-choice systems.

A finite *ordinary Petri net* is a pair (N, μ) : The *net* $N = (P, T, F)$ comprises a finite set P of *places*, a disjoint finite set T of *transitions* and a set $F \subseteq (P \times T) \cup (T \times P)$ of *directed arcs*. The function $\mu: P \rightarrow \mathbf{N}$ is named the *initial marking* of the net. The *support* of the marking μ is the set

$$supp(\mu) := \{ p \in P : \mu(p) > 0 \}$$

of all places marked at μ . All Petri nets in this paper will be assumed finite.

A *path* from a node $x_{ini} \in X := P \cup T$ to a node $x_{fin} \in X$ is a sequence (x_0, x_1, \dots, x_n) with nodes $x_i \in X$, $x_0 = x_{ini}$, $x_n = x_{fin}$ and $(x_i, x_{i+1}) \in F$. It is named *elementary path*, if $x_i \neq x_j$ for all pairs $i \neq j$. The net N is *strongly connected* if for every two nodes $x_1, x_2 \in X$ a path from x_1 to x_2 and a path from x_2 to x_1 exists.

A transition with a single pre-place and two or more post-places is an *opening* transition, a transition with a single post-place and two or more pre-places is called a *closing* transition. Opening transitions with exactly two post-places and closing transitions with exactly two pre-places are called *binary* transitions. A net N is called *binary* if all its transitions are binary.

For a net N the *firing rule* defines the firing of a transition: A transition $t \in T$ is *enabled* at a marking μ of N iff each place from its pre-set $pre(t)$ is marked at μ with at least one token. Being enabled, t may *occur* or *fire*. Firing t yields a new marking μ' , which results from μ by consuming one token from each pre-place of t and by producing one additional token on each post-place of t ; this is denoted by $\mu \xrightarrow{t} \mu'$.

A finite *occurrence sequence* from μ is a sequence $\sigma = t_1 \dots t_k$, $k \in \mathbf{N}$, such that

$$\mu \xrightarrow{t_1} \mu_1, \dots, \mu_{k-1} \xrightarrow{t_k} \mu_k.$$

We denote by $\mu \xrightarrow{\sigma} \mu_k$ the fact, that firing σ yields the marking μ_k . A *reachable marking* of a Petri net (N, μ) is a marking, which results from firing a finite occurrence sequence from μ . If not stated the contrary, occurrence sequences in this paper will be considered finite occurrence sequences. The *concatenation of two occurrence sequences* σ_1 and σ_2 is denoted by $\sigma_1 \cdot \sigma_2$.

A Petri net (N, μ_0) is *live* iff for each reachable marking μ and for each transition $t \in T$ the Petri net (N, μ) has a reachable marking which enables t . A Petri net is *k-bounded* iff a number $k \in \mathbb{N}$ exists bounding from above the token content of every place at every reachable marking. If the bound can be chosen as $k=1$ then the Petri net is named *safe*. A live and safe Petri net is named *well-behaved*. A net N is *well-formed* iff there exists a marking μ_0 of N such that the Petri net (N, μ_0) is live and bounded.

We will often dispense with an explicit notation for the set of places and transitions of a net and use the shorthand $x \in N$ to denote a node of the net.

1. Definition (P-system, T-system, free-choice system)

i) A net N is a *P-net* if all transitions have exactly one pre-place and exactly one post-place, i.e.

$$\text{card}[pre(t)] = 1 = \text{card}[post(t)] \text{ for all transitions } t \in N.$$

A *P-system* is a Petri net (N, μ) with N a P-net.

ii) A net N is a *T-net* if all places have exactly one pre-transition and exactly one post-transition, i.e.

$$\text{card}[pre(p)] = 1 = \text{card}[post(p)] \text{ for all places } p \in N.$$

A *T-system* is a Petri net (N, μ) with N a T-net.

iii) A net N is a *free-choice net* if for every two transitions $t_1, t_2 \in N$

$$\text{either } pre(t_1) \cap pre(t_2) = \emptyset \text{ or } pre(t_1) = pre(t_2).$$

A *restricted free-choice net* is a net which satisfies the stronger condition: For every two transitions $t_1, t_2 \in T$

$$\text{either } pre(t_1) \cap pre(t_2) = \emptyset \text{ or } pre(t_1) = pre(t_2) = \{ p \}$$

with a single place $p \in P$. A marked (restricted) free-choice net (N, μ) is named *(restricted) free-choice system*.

Well-behaved free-choice systems are one of the two classes of Petri nets studied in the present paper. By a theorem of Genrich each well-formed free-choice net FN has a marking μ , such that (FN, μ) is even well-behaved ([De1995] Theor. 5.10).

An important means for the examination of free-choice systems is the study of their P-components and T-components.

2. Definition (Components and their intersection)

Consider a net $N = (P, T, F)$.

i) A subnet $N_p \subseteq N$ which is generated by a nonempty subset $X \subseteq P \cup T$ of nodes, is a *P-component* of N if N_p is a strongly connected P-net with

$$pre(p) \cup post(p) \subseteq X \text{ for all places } p \in X .$$

Consider a marking μ of N . If a P-component $N_p \subseteq N$ is marked at μ with a single token then $(N_p, \mu_p), \mu_p := \mu \upharpoonright N_p$, is named a *basic component* of (N, μ) .

ii) A subnet N_T of N which is generated by a nonempty subset $X \subseteq P \cup T$ of nodes, is a *T-component* of N if N_T is a strongly connected T-net with

$$pre(t) \cup post(t) \subseteq X \text{ for all transitions } t \in X .$$

iii) The net N is *structurally non-blocking* iff every P-component N_p of N intersects every T-component N_T of N in a non-empty set $N_p \cap N_T \neq \emptyset$. Otherwise the net is named *structurally blocking*. A Petri net (N, μ) is *non-blocking* if its underlying net N is structurally non-blocking. Otherwise the Petri net is named *blocking*.

3. Example (Well-behaved, but blocking free-choice system)

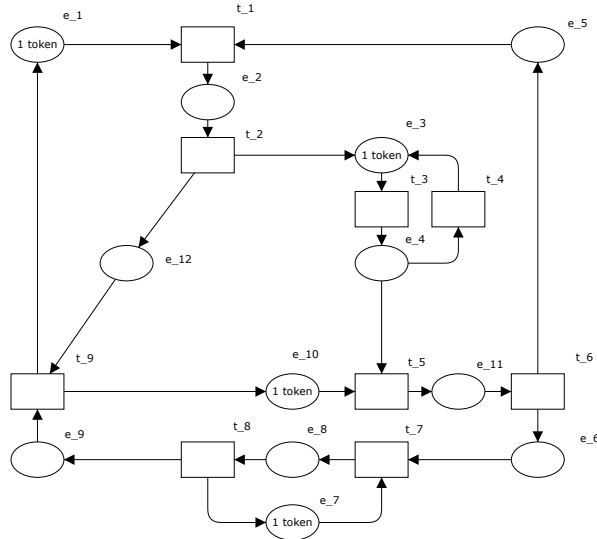


Figure 1: Well-behaved free-choice system FS

Figure 1 shows a well-behaved restricted free-choice system FS . It is blocking: E.g., the P-component N_P generated by the set $\{e_7, t_7, e_8, t_8\}$ and the T-component N_T generated by the set $\{e_3, t_3, e_4, t_4\}$ are disjoint.

It is well-known that a well-behaved free-choice system is covered by basic components. In the reverse direction, a Petri net is safe if it is covered by basic components. The following Lemma prepares the introduction of the new concept of non-blocking components.

4. Lemma (Well-behavedness of a union of basic components)

For a well-behaved free-choice system $FS = (FN, \mu)$ any union

$$(BN, \mu_{BN}) := \left(\bigcup_{i \in I} BN_i, \sum_{i \in I} \mu_i \right)$$

of basic components $BS_i = (BN_i, \mu_i)$, $\mu_i := \mu \upharpoonright BN_i$, of FS is a well-behaved free-choice system, too.

Proof. i) Apparently the net BN is free-choice.

ii) We derive the liveness of (BN, μ_{BN}) from Commoners Theorem, cf. [DE1995], Theor. 4.21 and 4.27. Because $BN \subseteq FN$ is transition bounded a given siphon R_{siph} of BN is also a siphon of FN . Because FS is live, a trap $R_{trap} \subseteq R_{siph}$ of FN exists which is marked at μ . Due to $R_{trap} \subseteq BN$ the set $R_{trap} \subseteq R_{siph}$ is also a trap of BN and is marked at μ_{BN} .

iii) All basic components $BS_i \subseteq FS, i \in I$, are also basic components of (BN, μ_{BN}) . Therefore (BN, μ_{BN}) is safe because it is covered by basic components, q. e. d.

The first new concept of this paper is the concept of a *non-blocking component*. It is a certain maximal, well-behaved and non-blocking subsystem of a well-behaved free-choice system.

5. Definition (Non-blocking component)

Consider a well-behaved free-choice system $FS = (N, \mu)$.

i) For a connected subnet $NB \subseteq N$ the restriction

$$NS := (NB, \mu_B), \mu_B := \mu \upharpoonright NB,$$

is named a *non-blocking component* of FS , iff NS is

- a union of basic components of FS and
- non-blocking and

- maximal with respect to these two properties, i.e. no subsystem of FS exists with these properties and containing NS as a proper subsystem.
- ii) A family $(NS_i)_{i \in I}$ of non-blocking components NS_i of FS , $i \in I$, with

$$FS = \bigcup_{i \in I} NS_i$$

is named a *non-blocking covering* of FS .

Apparently any well-behaved free-choice system has a non-blocking covering because each basic component is non-blocking. In addition, each non-blocking component of FS is well-behaved due to Lemma 4. This will be a crucial means for the construction in Definition 13.

A covering of a free-choice system is named *unshortenable* if no proper subfamily is a covering too. Each covering contains an unshortenable covering as a subfamily: After successively cancelling covering elements contained in the union of other elements we eventually obtain an unshortenable covering.

6. Example (Non-blocking covering)

The well-behaved blocking free-choice system FS from Figure 1 has an unshortenable non-blocking covering with two non-blocking components, cf. Figure 2. One non-blocking component is the union of three different basic components while the other non-blocking component is a single basic component.

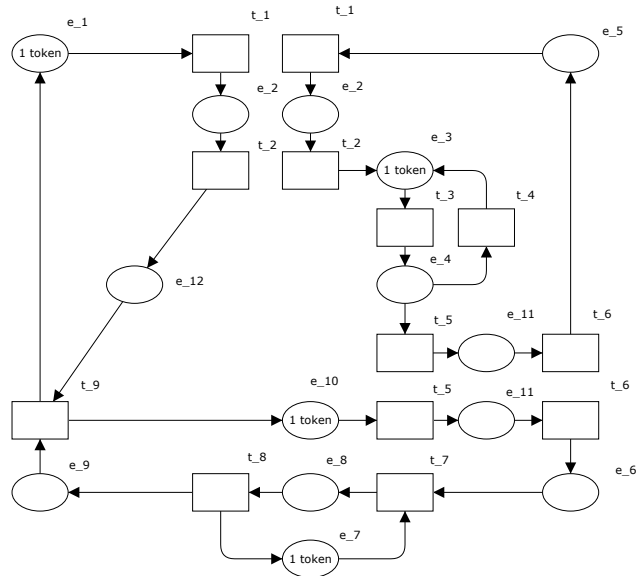


Figure 2: Two non-blocking components of the free-choice system from Figure 1

3. Linked bp-systems

Bp-systems are a simple class of coloured Petri nets. As mentioned in the Introduction they can be used to define the Boolean semantics of AND/XOR-EPCs. For the present paper we do not need the concept of coloured Petri nets in full generality, the interested reader is referred to [Jen1992].

7. Definition (bp-system)

i) A *bipolar synchronization graph* (bp-graph) BG is a coloured net. It extends a T-net $N = (P, T, F)$ by attaching to each place $p \in P$ the fixed set

$$C(p) = Boole := \{ high, low \}$$

with two token colours and provides each transition $t \in T$ with one from two types of logic:

- An AND-transition $t = t_{AND}$ has a set of firing modes $B(t) = \{ high, low \}$ with two elements: The high mode (respectively low mode) is enabled iff all pre-places of t_{AND} are marked with at least one high token (respectively low token). Its firing consumes one high token (respectively low token) from each pre-place and creates one high token (respectively low token) on every post-place.
- An XOR-transition $t = t_{XOR}$ with n pre-places and m post-places has a set of firing modes $B(t) = \{ b_{(i,j)} \}$ with $n \cdot m$ high modes and one low mode: The high mode with index (i, j) , $1 \leq i \leq n, 1 \leq j \leq m$, is enabled iff the i -th pre-place is marked with at least one high token and all other pre-places with at least one low token. Firing the high mode consumes a high token from the i -th pre-place and a low token from every other pre-place and creates a high token at the j -th post-place and a low token at every other post-place. The low mode is enabled iff all pre-places are marked with at least one low token. Firing the low mode consumes a low token from each pre-place and creates a low token at every post-place.

Adhering to the common notation of coloured nets we call a pair (p, c) with $p \in P, c \in C(p)$, a *token element* and a pair (t, b) with $t \in T, b \in B(t)$, a *binding element*. A binding element is named *low binding element*, if its firing consumes and creates only low tokens. Otherwise it is named *high binding element*.

ii) A *bipolar synchronization system* (bp-system) is a coloured Petri net $BS = (BG, \mu)$ with a bp-graph BG and an initial marking μ with at least one high token.

Bp-systems are a special case of Boolean systems which have been introduced in [LSW1998].

8. Definition (Well-behavedness of a bp-system)

i) A bp-system BS is *safe* iff each reachable marking marks every place with at most one token.

- ii) A binding element of a bp-system BS is *live* iff for every reachable marking μ_1 of BS the bp-system (BG, μ_1) has a reachable marking which enables the given binding element. BS is *live with respect to all its high bindings* iff every high binding element of BS is live.
- iii) A bp-system BS is *well-behaved* iff it is safe and live with respect to all its high bindings.

9. Remark (Morphisms of Petri nets)

Within the category of coloured Petri nets the concept of a morphism

$$f : PN_1 \longrightarrow PN_2$$

between two coloured Petri nets is well-defined, cf. [Weh2006]. Our concept of a morphism presupposes coloured nets for the domain and range of the morphism, because a morphism maps respectively, certain T-flows and P-flows of PN_1 to binding elements and token elements of PN_2 .

The reader, who is not interested in the general definition of a morphism, may use his own descriptive concept of a morphism $PN_1 \xrightarrow{f} PN_2$ to follow the examples of this paper. In most cases the domain of definition PN_1 will be an ordinary Petri net and the coloured Petri net PN_2 will be equivalent to an ordinary Petri net, too. In addition, all morphisms under consideration will be *discrete*, i.e. for any node $y \in PN_2$ the fibre $f^{-1}(y) \subset PN_1$ has only isolated nodes.

In a previous paper [Weh2010], Chap. 2, we have attached several ordinary Petri nets to a given bp-system $BS = (BG, \mu)$. Notably, a bp-system BS has a restricted free choice system

$$BS^{high} = (BG^{high}, \mu^{high}),$$

the *high-system* of BS , together with a morphism $high : BS \longrightarrow BS^{high}$ as well as a T-system

$$BS^{skel} = (BG^{skel}, \mu^{skel}),$$

the *skeleton* of BS , together with a morphism $skel : BS \longrightarrow BS^{skel}$.

Conversely, each restricted free-choice system FS extends to a bp-system BS with $BS^{high} = FS$. Hereby one introduces an AND-transition of BS for a branched transition of FS , an XOR-transition of BS for a branched place of FS and a high token of BS for each token of FS .

10. Example (Well-behaved bp-systems)

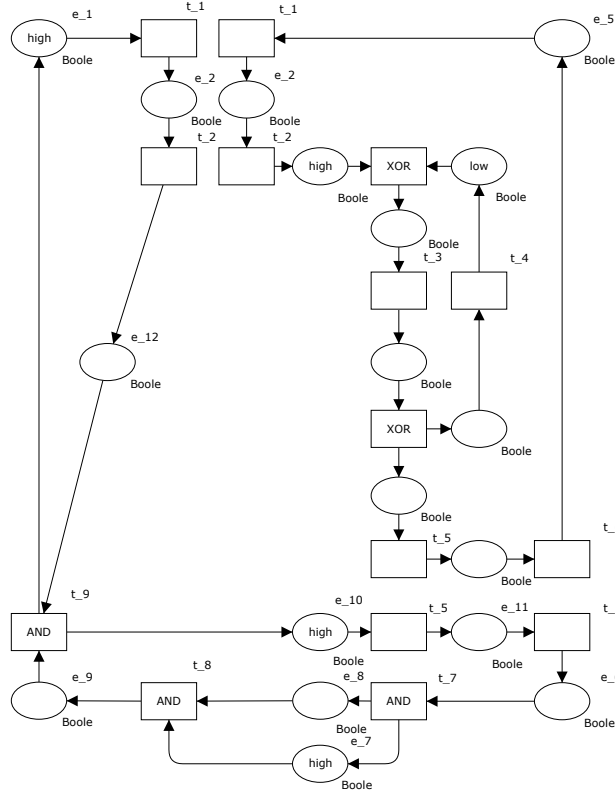


Figure 3: Bp-systems of the non-blocking components from Figure 2

Figure 3 shows two bp-systems $BS_i, i=1,2$. Both are well-behaved. Their high-systems BS_i^{high} are the two non-blocking components from the well-behaved free-choice system from Figure 2. Note the low token marking the post-place of transition t_4 .

Any discrete Petri net morphism $PN_1 \xrightarrow{f} PN_2$ maps occurrence sequences of PN_1 to occurrence sequences of PN_2 . The question about the surjectivity of this map is named the lifting problem.

11. Definition (Lifting property of a morphism)

A Petri net morphism

$$PN_1 \xrightarrow{f} PN_2$$

has the *lifting property* iff for any enabled occurrence sequence σ_2 of PN_2 an enabled occurrence sequence σ_1 of PN_1 exists with $f(\sigma_1) = \sigma_2$. The occurrence sequence σ_1 is named a *lift of σ_2 against f* .

The following Proposition 12 shows the relation between well-behaved bp-systems and well-behaved restricted free-choice systems. The proposition has been proven in [Weh2010].

12. Proposition (Bipolar systems and non-blocking free-choice systems)

- i) A bp-system is well-behaved iff its skeleton and its high system are well-behaved and its high-system is non-blocking.
- ii) The high-morphism $high : BS \longrightarrow BS^{high}$ of a well-behaved bp-system BS has the lifting property.
- iii) Any well-behaved, non-blocking restricted free-choice system FS is the high-system of a well-behaved bp-system BS .

To obtain a marking such that the bp-scheme BS in Proposition 12, iii) is well-behaved, possibly some low tokens have to be added in addition to the high tokens prescribed by the marking of FS .

On the other hand, if a restricted free-choice system FS is well-behaved but blocking, no well-behaved bp-system BS exists with $BS^{high} = FS$.

It is our particular concern in this paper to remedy this situation. Therefore we will apply Proposition 12, iii) separately for each element from a non-blocking covering $(FS_i)_{i \in I}$ of FS : For each non-blocking component FS_i we obtain a well-behaved bp-system BFS_i with $BFS_i^{high} = FS_i$. For each pair of bp-systems (BFS_i, BFS_j) we fuse those subsystems of BFS_i and BFS_j which project along the high morphisms onto the same subsystem of FS . In the Petri net BS_i , which results from BFS_i , we consider the low tokens from BFS_i to belong to BS_i exclusively. For each pair (FS_i, FS_j) of non-blocking components of FS we substitute each branched transition $t^{high} \in \partial(FS_i \cap FS_j) \subset FS$ from the boundary by a transition which fuses the corresponding bp-systems (BS_i, BS_j) . The fusing transition has to satisfy the following requirements:

- When firing it consumes and creates high tokens from BS_i and BS_j in the same manner as t^{high} processes tokens from FS_i and FS_j .

- It consumes and creates low tokens from BS_i without synchronizing them with high tokens or with low tokens from BS_j . Analogously it consumes and creates low tokens from BS_j .

The resulting coloured Petri net is named a *linked bipolar system* (bp-system). It is the second new concept introduced in this paper.

13. Definition (Linked bp-system)

Consider a well-behaved free-choice system FS and a covering $(FS_i)_{i \in I}$ of FS by non-blocking components. According to Lemma 4 each non-blocking component FS_i , $i \in I$ is a well-behaved free-choice system. It is the high-system of a well-behaved bp-system BFS_i , and the high-morphism $high_i : BFS_i \longrightarrow FS_i$ has the lifting property according to Proposition 12. We define a coloured Petri net

$$LBS := \frac{\dot{\bigcup}_{i \in I} BFS_i}{(high_i)_{i \in I}}$$

by forming the quotient of the disjoint union of the bp-systems BFS_i , $i \in I$, modulo the identification with respect to the family of high morphisms $high_i : BFS_i \longrightarrow FS_i$. The coloured Petri net LBS is named a *linked bipolar-system (bp-system) attached to FS with respect to the covering FS_i , $i \in I$* . The high morphisms induce a well-defined morphism of Petri nets

$$high : SBS \longrightarrow FS .$$

Note that for each index $i \in I$ a projection $\pi_i : BFS_i \longrightarrow SBS$ onto the quotient exists. The image is a transition bounded subsystem

$$BS_i := \pi_i(BFS_i) \subset SBS .$$

Using the notations

$$\begin{aligned} SBS &= (SBN, \mu), \quad BFS_i = (BFN_i, \mu_i), \quad BS_i = (BN_i, \mu_i), \\ FS &= (FN, \nu), \quad FS_i = (FN_i, \nu_i) \end{aligned}$$

Definition 13 of a linked bp-system SBS can be made explicit as follows:

- Nodes SBS : Two nodes $x_i \in BFX_i$ and $x_j \in BFX_j$ fuse to a node $x \in SBS$ iff $high_i(x_i) = high_j(x_j) \in (FX_i \cap FX_j) \subset FX$. A well-defined map $high : SBS \longrightarrow FX$ results. We define

$$I(x) := \{ i \in I : x \text{ has a representative } x_i \in BFX_i \}.$$

- Token colours of LBN : A place $p \in LBX$ gets the set of token colours

$$C(p) := \{ high \} \cup \{ low_i : i \in I(p) \} .$$

- Bindings and firing rules of LBN : For a transition $t \in LBX$ the binding set $B(t)$ has as low bindings the low bindings of all representatives $t_i \in BFN_i, i \in I(t)$, of t , each taken with its firing rule. On the other hand, the high bindings in $B(t)$ correspond bijectively to the high bindings of one arbitrary t_i . Each high binding gets an unchanged flow of high tokens. If all representatives t_i have logical type AND, then the firing rule of a high binding of t does not consider any low tokens. When all representatives t_i have logical type XOR, then the firing rule of a high binding may change the flow of low tokens: When a high binding of t_{i_0} consumes a single low token of type low_{i_0} at a pre-place of t_{i_0} or creates a single low token of type low_{i_0} at a post-place, then the corresponding high binding from $B(t)$ respectively consumes and creates all low tokens of type $low_i, i \in I(t)$, at the corresponding place of LBN .
- Initial marking of LBS : At a place $p \in LBN$ the initial marking μ is defined as

$$\mu(p) := v(high(p)) + \sum_{i \in I(p)} \mu_i^{low}(p_i) \in C(p)_N .$$

- High morphism: For each index $i \in I$ the morphism $high_i : BFS_i \longrightarrow FS_i$ induces a morphism $high_i : BS_i \longrightarrow FS_i$ from the quotient $BS_i := \pi_i(BFS_i)$. These local morphisms fuse to a global morphism $high : LBS \longrightarrow FS$, such that

$$\begin{array}{ccc} LBS & \longrightarrow & LBS_i \\ \downarrow high & & \downarrow high_i \\ FS & \longrightarrow & FS_i \end{array}$$

commutes for all $i \in I$, the horizontal maps being the restrictions onto closed subsystems.

Figure 4 displays the most simple case of linking two binary bp-systems $BFS_i, i = 1, 2$, at a transition $t \in \partial(BN_1 \cap BN_2)$. The token colours of its pre- and post-places are

$$Boole_i = \{ high, low_i \}, i = 1, 2, \text{ and } Boole_{12} = \{ high, low_1, low_2 \}.$$

On the left is shown the neighbourhood of a fused closing transition of logical type UNITE and on the right of an opening transition of logical type SEP (separate).

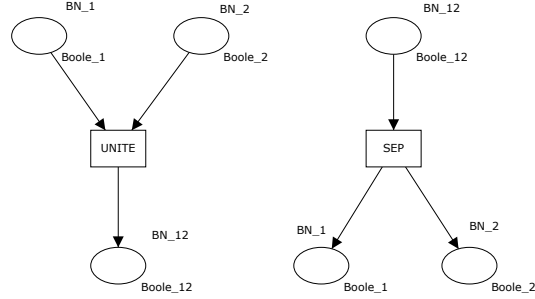


Figure 4: Transitions of type UNITE and SEP

Table 1 shows the corresponding firing rules of all transitions from $BN_{12} := BN_1 \cap BN_2$. Note in particular the different types of low tokens and their flow.

Transition	Bindings	consumes	produces
$t \in \partial(BN_1 \cap BN_2)$ of type SEP	<i>high</i>	<i>high</i>	$(high, high)$
	low_1	low_1	$(low_1, -)$
	low_2	low_2	$(-, low_2)$
UNITE: reverse SEP			
$t \in (BN_1 \cap BN_2)^\circ$ from opening XOR	$high_{left}$	<i>high</i>	$(high, low_1 + low_2)$
	$high_{right}$	<i>high</i>	$(low_1 + low_2, high)$
	low_1	low_1	(low_1, low_1)
	low_2	low_2	(low_2, low_2)
closing XOR: reverse opening XOR			
$t \in (BN_1 \cap BN_2)^\circ$ from opening AND	<i>high</i>	<i>high</i>	$(high, high)$
	low_1	low_1	(low_1, low_1)
	low_2	low_2	(low_2, low_2)
closing AND: reverse opening AND			

Table 1: Binding elements of BN_{12}

14. Example (Linked bp-system)

Figure 5 shows the linked bp-system attached to the well-behaved free-choice system from Example 3 and its non-blocking covering from Example 6.

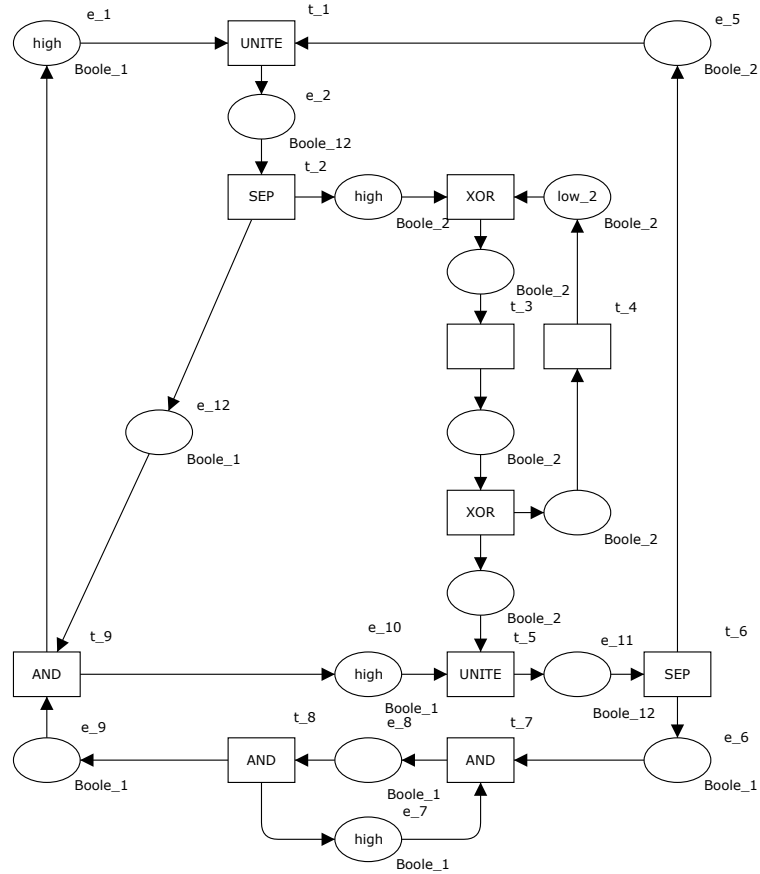


Figure 5: Linked bp-system attached to the free-choice system from Figure 1

15. Remark (Non-uniqueness of a linked bp-system)

According to the construction from Definition 13 a well-behaved free-choice system $FS = (FN, \nu)$ has more than one linked bp-system $LBS = (LBN, \mu)$ in general.

A first reason for non-uniqueness is the choice of a non-blocking covering $\mathcal{N} = (NS_i)_{i \in I}$ of FS . In general FS has more than one non-blocking covering which is non-shortenable. Therefore the underlying net $LBN = LBN(FS, \mathcal{N})$ depends not only on FS but also on \mathcal{N} .

This kind of dependency is similar to other situations from mathematics. E.g., compare the definition of a differentiable manifold, which is considered a pair (X, \mathcal{A}) formed by a topological space X and a maximal differentiable atlas \mathcal{A} on X . But different from the situation of differentiable manifolds two non-blocking coverings \mathcal{N}_1 and \mathcal{N}_2 of a well-behaved free-choice system FS are compatible with each other: Their union $\mathcal{N}_1 \cup \mathcal{N}_2$ is a non-blocking covering of FS again. Employing the definition of morphisms between coloured Petri nets from [Weh2006] one can show that the net of the corresponding linked bp-system is the fibre product

$$LBN(FS, \mathcal{N}_1 \cup \mathcal{N}_2) = LBN(FS, \mathcal{N}_1) \times_{FS} LBN(FS, \mathcal{N}_2)$$

with respect to the high-morphisms

$$high_i : LBN(FS, \mathcal{N}_i) \longrightarrow FN, i = 1, 2.$$

As a consequence the covering \mathcal{N}_{\max} formed by all non-blocking components of FS is the unique maximal non-blocking covering of FS and one can define $LBN(FS, \mathcal{N}_{\max})$ as the underlying net of any linked bp-system of FS .

A second reason for non-uniqueness is the choice of the low tokens when considering a fixed non-blocking component FS_i of FS . In general more than one marking μ_i exists with $BFS_i = (BFN_i, \mu_i)$ well-behaved and $BFS_i^{high} = FS_i$. Two different markings differ by the distribution of low tokens. As a consequence, there may exist more than one marking μ on $LBN = LBN(FS, \mathcal{N}_{\max})$ with $LBS = (LBN, \mu)$ a linked bp-system of FS .

The following Theorem 16 and its corollary Proposition 18 are the main results of the present paper. They prove that any linked bp-system of a well-behaved free-choice system is well-behaved too.

16. Theorem (High morphism of a linked bp-system)

Consider a well-behaved free-choice system FS and a non-blocking covering $(FS_i)_{i \in I}$ of FS . The high morphism

$$high : LBS \longrightarrow FS$$

from a linked bp-system LBS attached to FS with respect to $(FS_i)_{i \in I}$ has the lifting property.

Proof. We use the following notations from Definition 13

$$LBS = (LBN, \mu), BFS_i = (BFN_i, \mu_i), FS = (FN, \mu^{high}), FS_i = (FN_i, \mu_i^{high})$$

and denote by $pr_i : FN \longrightarrow FN_i$ the canonical projection.

In order to prove the lifting property of $high: LBS \longrightarrow FS$ we start considering an occurrence sequence

$$\mu^{high} \xrightarrow{\sigma^{high}} \nu^{high}$$

of FS . Without loss of generality we may assume that σ^{high} comprises only a single transition $r \in FN$. For each index $i \in I$ we define the transition $r_i := pr_i(r) \in FN_i$. The condition $high(t, b) = r$ determines a unique transition $t \in LBN$ and a unique high binding $b \in B(t)$ of t . Analogously, for each index $i \in I$ the condition $high_i(t_i, b_i) = r_i$ determines a unique transition $t_i \in BFN_i$ and a unique high binding $b_i \in B(t_i)$ of t_i . In addition, according to Proposition 12 an occurrence sequence $\mu_i \xrightarrow{\sigma_i^{low}} \tilde{\mu}_i$ exists in the low-system BFS_i^{low} such that the marking $\tilde{\mu}_i$ of BFS_i activates the binding element (t_i, b_i) . By catenation we obtain an occurrence sequence

$$\mu_i \xrightarrow{\sigma_i} \nu_i$$

of BFS_i with $\sigma_i := \sigma_i^{low} \cdot (t_i, b_i)$. Because $BS_i^{low} \subset LBS$ is a place bounded subsystem the occurrence sequence $\pi_i(\sigma_i^{low})$ of BS_i^{low} can be considered an enabled occurrence sequence of BS . Firing $\pi_j(\sigma_j^{low})$ for an arbitrary index $j \neq i$ does not remove the firing concession from $\pi_i(\sigma_i^{low})$. By catenation we obtain an occurrence sequence of LBS

$$\mu \xrightarrow{\sigma^{low}} \tilde{\mu} \text{ with } \sigma^{low} := \pi_1(\sigma_1^{low}) \cdot \dots \cdot \pi_n(\sigma_n^{low})$$

Due to Definition 13 the occurrence sequences $\tilde{\mu}_i \xrightarrow{(t_i, b_i)} \nu_i$, $i \in I$, of all bp-systems BFS_i link to an occurrence sequence $\tilde{\mu} \xrightarrow{(t, b)} \nu$ of LBS . By catenating $\sigma := \sigma^{low} \cdot (t, b)$ we obtain the occurrence sequence of LBS sought-after

$$\mu \xrightarrow{\sigma} \nu$$

satisfying $high(\sigma) = \sigma^{high}$, q.e. d.

For a linked bp-system the definition of *liveness with respect to all its high bindings* is literally the same as in Definition 8, part ii) for a bp-system.

17. Definition (Well-behavedness of linked bp-systems)

Consider a linked bp-system LBS attached to a well-behaved free-choice system FS and a non-blocking covering of FS with $k \geq 1$ elements.

- i) LBS is *high-safe* iff every reachable marking of LBS marks each place either with a single high token but no low token or with at most k low tokens but no high token.
- ii) LBS is *well-behaved* iff it is high-safe and live with respect to all its high bindings.

18. Proposition (Well-behavedness of a linked bp-system)

Any linked bp-system LBS attached to a well-behaved free-choice system FS with respect to a non-blocking covering $(FS_i)_{i \in I}$ is well-behaved.

Proof. With the notations of Definition 13 we set

$$LBS = (LBN, \mu_0), \quad FS = (FN, high(\mu_0)) = LBS^{high} \quad \text{and} \quad (BFS_i)_{i \in I}.$$

- i) High-safeness of LBS follows from the existence of the morphism $high : LBS \longrightarrow FS$ and the safeness of each $BFS_i, i \in I$.
- ii) For the proof that LBS is live with respect to all high bindings we employ the lifting property of $high : LBS \longrightarrow FS$. We consider an occurrence sequence $\mu_0 \xrightarrow{\sigma_1} \mu_1$ and a transition $t \in LBN$ with a high binding $b \in B(t)$. By definition the binding element (t, b) of LBN is a transition of FN . Liveness of FS implies the existence of an occurrence sequence

$$high(\mu_1) \xrightarrow{\sigma_2^{high}} \mu_2^{high}$$

such that the marking μ_2^{high} activates (t, b) . According to Theorem 16 the occurrence sequence σ_2^{high} lifts to an occurrence sequence $\mu_1 \xrightarrow{\sigma_2} \mu_2$ such that the following diagram commutes

$$\begin{array}{ccccc} \mu_0 & \xrightarrow{\sigma_1} & \mu_1 & \xrightarrow{\sigma_2} & \mu_2 \\ \downarrow high & & \downarrow high & & \downarrow high \\ high(\mu_0) & \xrightarrow{high(\sigma_1)} & high(\mu_1) & \xrightarrow{\sigma_2^{high} = high(\sigma_2)} & \mu_2^{high} = high(\mu_2) \end{array}$$

Therefore LBS is live with respect to all high bindings, q. e. d.

19. Remark (Semantics of AND/XOR-EPCs)

Let EPC be an AND/XOR-EPC. As described in the *Introduction* a free-choice system FS exists, which defines the free-choice semantics of EPC . Also a translation of EPC into a bp-system BS exists. We have $BS^{high} = FS$.

- i) If FS is non-blocking, then BS is well-behaved iff FS is well-behaved, cf. Proposition 12.

- ii) If FS is well-behaved, then the Boolean semantics of EPC is defined by a well-behaved linked bp-system LBS with $LBS^{high} = FS$, cf. Proposition 18.
- iii) If FS is non-blocking and well-behaved then $LBS = BS$, cf. Definition 13.

4. Outlook

We have shown that any AND/XOR-EPC, which is well-behaved with respect to its free-choice semantics, can be provided with a Boolean semantics which is well-behaved too. In order to obtain this result, we had to generalize bp-systems to linked bp-systems, a class of coloured Petri nets which is slightly more general.

This step is necessary to tackle EPC with connectors of arbitrary logical type. Apparently one can translate every EPC literally into a Boolean system. But Example 14 indicates that the literal translation possibly has to be altered afterwards to avoid the blocking of low tokens.

Future investigations have to consider the literal translation of an EPC into a Boolean system only as a starting point. For the next step we need an algorithm which identifies well-behaved components of the Boolean system. Then it should link these components to a linked Boolean system, which avoids the blocking of low tokens from different components. These well-behaved components generalize the bp-systems of non-blocking components while linked Boolean systems generalize the concept of linked bp-systems introduced in the present paper.

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