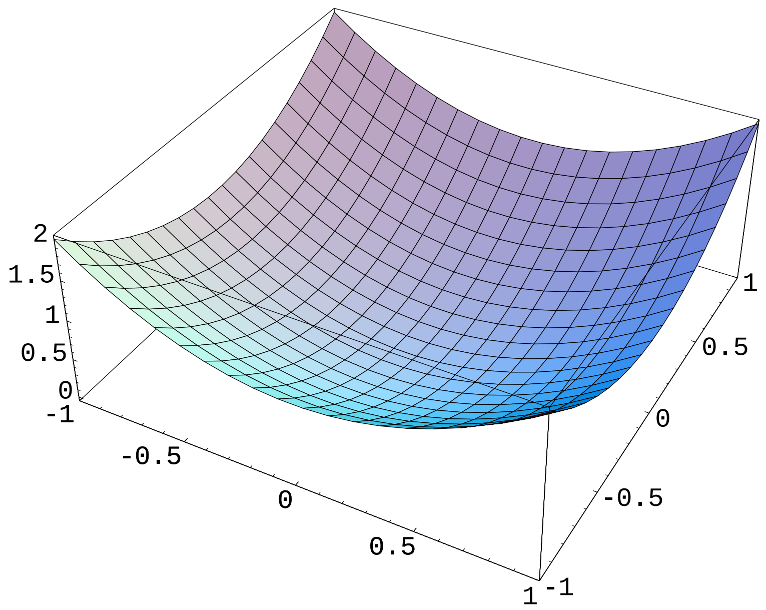
Lokales Minimum von $f(x,y) = x^2 + y^2$ bei $(0,0)$

Es gilt:

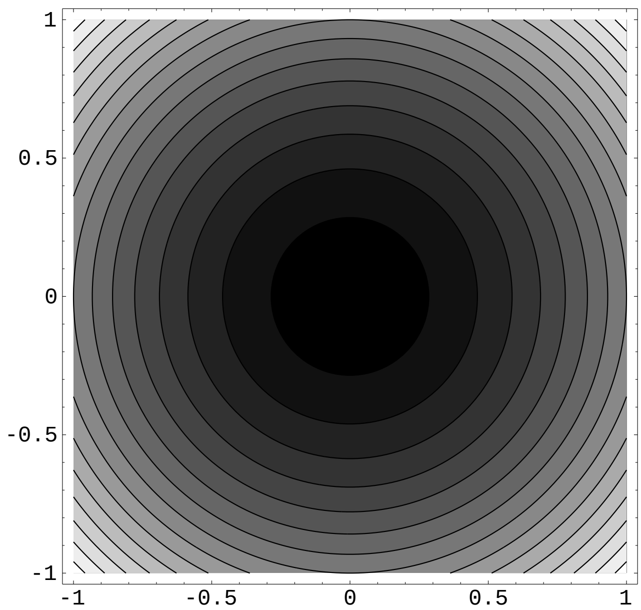
$$f(0,0) = 0, \quad df(0,0) = (0,0), \quad \text{Hess}\, f(0,0) = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

Die Hessematrix ist positiv definit.

- 3D-Plot:



- Niveaulinien:



("dunkel" $\equiv$ "kleine Werte")

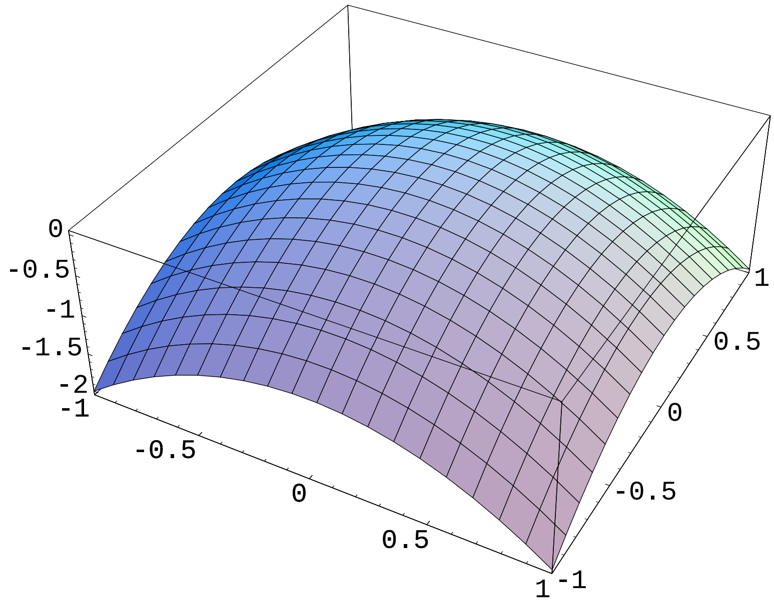
Lokales Maximum von $g(x,y) = -x^2 - y^2$ bei $(0,0)$

Es gilt:

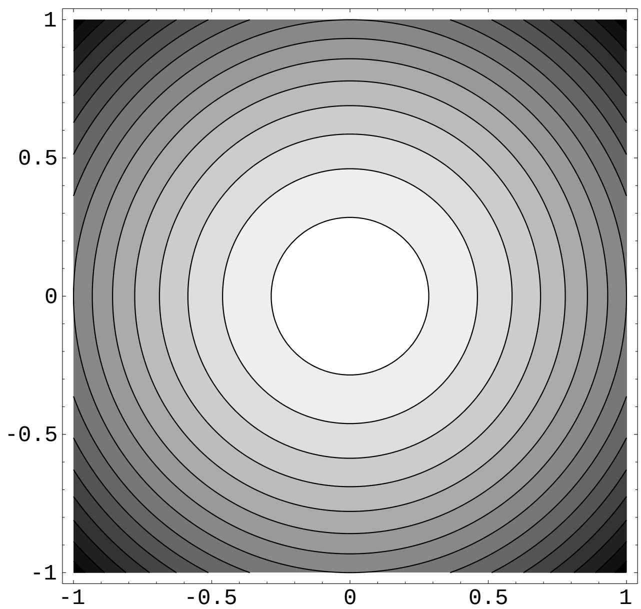
$$g(0,0) = 0, \quad dg(0,0) = (0,0), \quad \mathrm{Hess}\, g(0,0) = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$$

Die Hessematrix ist negativ definit.

- 3D-Plot:



- Niveaulinien:



("dunkel" $\equiv$ "kleine Werte")

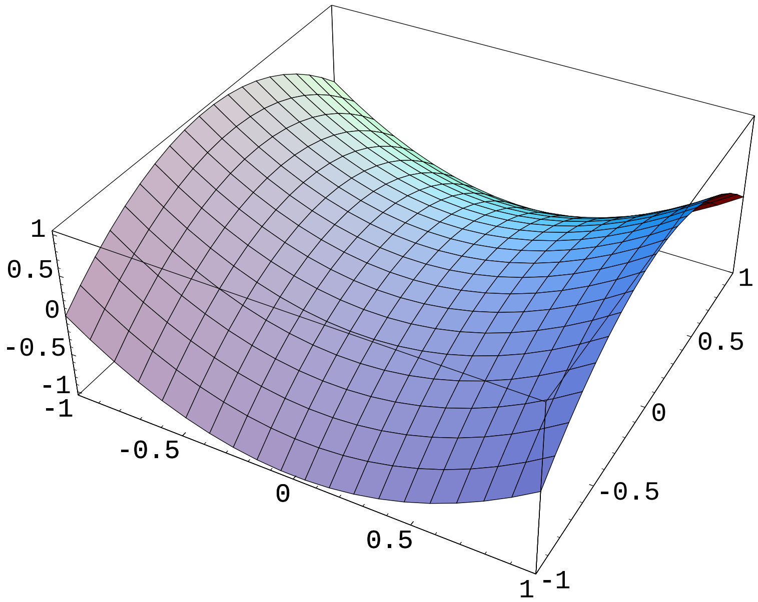
Sattelpunkt von $h(x,y) = x^2 - y^2$ bei $(0,0)$

Es gilt:

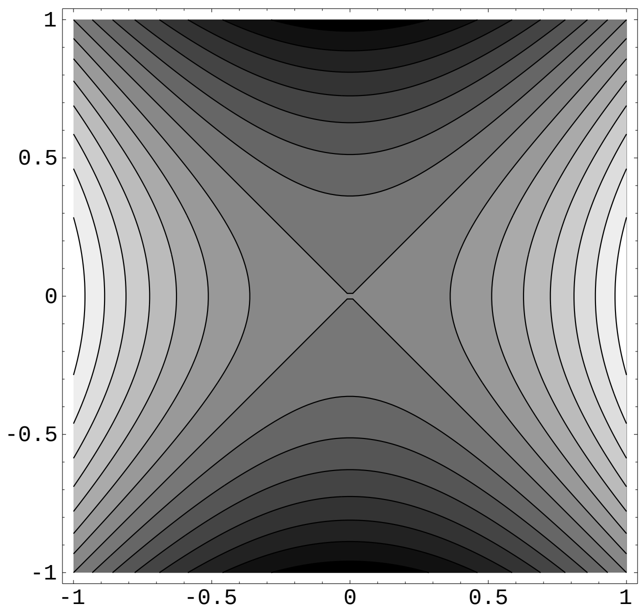
$$h(0,0) = 0, \quad dh(0,0) = (0,0), \quad \text{Hess}\, h(0,0) = \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}$$

Die Hessematrix ist indefinit.

- 3D-Plot:



- Niveaulinien:



("dunkel" $\equiv$ "kleine Werte")

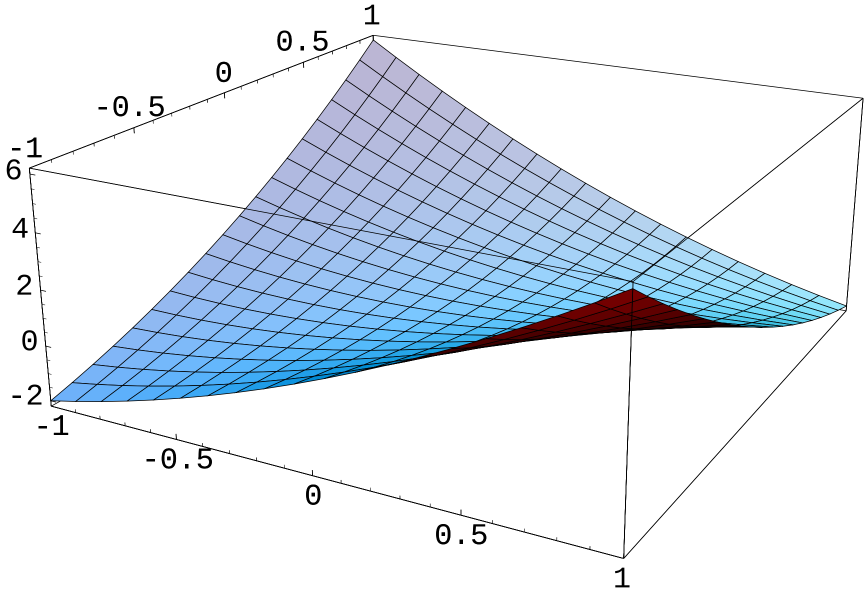
Sattelpunkt von $k(x,y) = x^2 + y^2 - 4xy$ **bei** $(0,0)$

Es gilt:

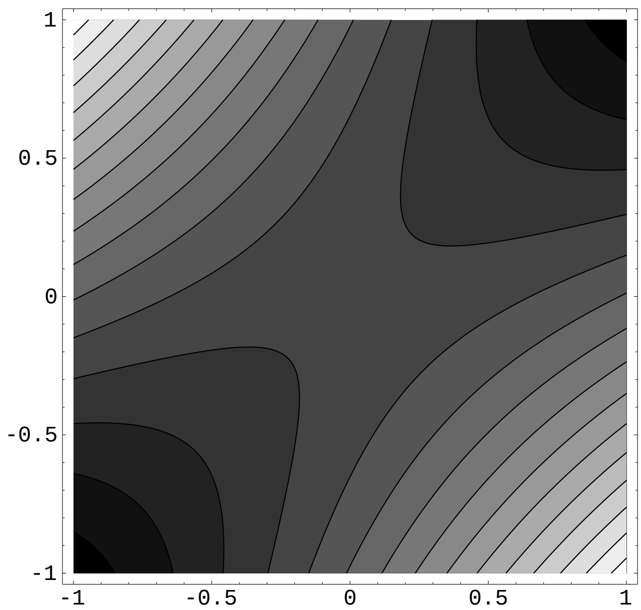
$$k(0,0) = 0, \quad dk(0,0) = (0,0), \quad \mathrm{Hess}\, k(0,0) = \begin{pmatrix} 2 & -4 \\ -4 & 2 \end{pmatrix}$$

Die Hessematrix ist indefinit, obwohl alle Diagonaleinträge positiv sind.

- 3D-Plot:



- Niveaulinien:



(“dunkel” $\equiv$ “kleine Werte”)