MATHEMATISCHES INSTITUT
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Numerik I
(Zentralübung)

Introductory Problem Sheet (Not for handing in)

The questions in this problem sheet are intended to provide a short recap on some material from analysis and linear algebra, which you may find useful as the lecture course progresses.

Question 1

Find the eigenvalues and corresponding eigenvectors of the following matrices.

$$A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 2 & -1 & 2 \\ 5 & -3 & 3 \\ -1 & 0 & -2 \end{pmatrix}.$$

Question 2

(a) Define the map $g\colon \mathbb{R}^2 \to \mathbb{R}^3$ by

$$g(x,y) := \begin{pmatrix} \sin x \\ \sin y \\ \sin(x+y) \end{pmatrix}.$$

Compute ∇g.

(b) Let $A \in \mathbb{R}^{n\times n}$ be a symmetric, positive definite $n \times n$ real Matrix, $b \in \mathbb{R}^n$, and $c \in \mathbb{R}$. Now define the real-valued function

$$f(x) := \frac{1}{2} x^t A x - x^t b + c\,.$$

Show that f has a single stationary point $x_0 \in \mathbb{R}^n$, given by

$$Ax_0 = b$$

or

$$x_0 = A^{-1} b\,.$$

Show also that x_0 is in fact a global minimum of f.

Question 3

Recall the following version of Taylor's Theorem, with Lagrange Remainder term:

If $f\colon [a,x] \to \mathbb{R}$ is a continuous function that is $k+1$-times differentiable on (a,x), then we have

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \ldots + \frac{f^{(k)}(a)}{k!}(x-a)^k + R_k(x)$$

where the remainder term $R_k(x)$ has the form

$$R_k(x) = \frac{f^{(k+1)}(c)}{(k+1)!}(x-a)^{k+1}$$

for some $c \in (a,x)$. The expansion above without the remainder term is often called the k^{th}-order Taylor Polynomial $P_k(x)$.

Now let $a = 0$, and for $x > 0$ let

$$f(x) = \log(1+x)\,.$$

($\log$ here denotes the natural logarithm). For $k \in \mathbb{N}$, compute the k^{th}-order Taylor Polynomial $P_k(x)$ expanded about 0 at x. For what $x > 0$ does the Remainder Term converge to zero as k tends to infinity? For what x does $P_k(x)$ converge as k tends to infinity?

Question 4

Prove that for $k \in \mathbb{N}$, $k \geq 2$, we have

$$\sum_{j=2}^{k} \frac{1}{j} \leq \log(k) \leq \sum_{j=1}^{k-1} \frac{1}{j}\,.$$

(*Hint:* Estimate the integral $\int_1^k 1/x \,\mathrm{d}x$ from above and below using functions that are constant on intervals $(j, j+1)$.)

Question 5

We will have a short introductory demonstration of MATLAB in class.