MATHEMATISCHES INSTITUT
DER UNIVERSITÄT MÜNCHEN
Prof. Dr. C. Seifert / Dr. P. Soneji

WS 2015/16
– Blatt 1 –
22.10.2015

Numerik I
(Zentralübung)

Problem Sheet 1

Question 1 **[4 x 1 = 4 points]**

Consider the space $\mathbb{R}^{n\times n}$ of real $n \times n$ matrices. Determine whether the following functionals $\|\cdot\|\colon \mathbb{R}^{n\times n} \to [0,+\infty)$ (where $A = (a_{i,j})_{1\le j\le n}^{1\le i\le j} \in \mathbb{R}^{n\times n}$) are norms on $\mathbb{R}^{n\times n}$ or not. Either show that they are norms, or explain why they are not.

(i) $\|A\| = \max_{1\le i,j\le n} |a_{i,j}|$

(ii) $\|A\| = |\det(A)|$

(iii) $\|A\| = \sqrt{\text{trace}(AA^t)}$

(iv) $\|A\| = \sqrt{\text{trace}(A^2)}$.

Question 2 **[2+2+3 points]**

Compute the absolute and relative condition numbers for $f\colon \mathbb{R} \to \mathbb{R}$ with respect to the absolute value (as norm) when

(a) $f(x) = e^{-x^2}$

(b) $f(x) = \ln(1 + x^2)$

(c) $f(x) = \frac{1-\cos(2x)}{2x}$.

Question 3 **[6 points]**

Let $n \in \mathbb{N}$ and $x_0, x_1, \dots, x_n \in \mathbb{R}$. Now let

$$V := \begin{pmatrix} 1 & x_0 & \cdots & x_0^n \\ 1 & x_1 & \cdots & x_1^n \\ \vdots & & \vdots & \\ 1 & x_n & \cdots & x_n^n \end{pmatrix} .$$

V is known as the *Vandermonde-Matrix*, and is important in the context of polynomial interpolation. Show that its determinant is given by

$$\det(V) = \prod_{i,j=0;i>j}^{n} (x_i - x_j).$$

Question 4 **[3 points]**

Show, by giving an example, that different matrix norms can induce different condition numbers for the same matrix.

Deadline for handing in: 1400 Thursday 29 October 2015

Please put solutions in one of the designated Numerik I boxes on the 1st floor (near the library)

Homepage: `www.mathematik.uni-muenchen.de/~soneji/numerik.php`