MATHEMATISCHES INSTITUT
DER UNIVERSITÄT MÜNCHEN
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WS 2014/15
– Blatt 11 –
17.12.2014

PDG I
(Zentralübung)

Problem Sheet 11

Question 1

Use the method of characteristics to determine the solution $u\colon [0,\infty)\times\mathbb{R}\to\mathbb{R}$ of the initial value problem

$$\begin{cases} u_t - xu_x = x & \text{in } (0,\infty)\times\mathbb{R} \\ u(0,x) = f(x) & \text{on } \{t=0\}\times\mathbb{R}\,, \end{cases}$$

where $f\in C^1(\mathbb{R})$ is given.

Question 2

Use the method of characteristics to find a solution $u\colon \mathbb{R}\setminus\{0\}\times\mathbb{R}\to\mathbb{R}$ $(=u(x,y))$ of the initial value problem

$$\begin{cases} (y+u)u_x + yu_y = x - y & \text{in } \mathbb{R}\setminus\{0\}\times\mathbb{R} \\ u(x,1) = 1 + x & \text{on } \mathbb{R}\setminus\{0\}\times\{y=1\}\,. \end{cases}$$

Question 3

(a) Use the method of characteristics to find a solution $u = u(x,y)$ to the Cauchy problem

$$\begin{cases} x^2u_x + y^2u_y = u^2 \\ u(x,2x) = x^2 \end{cases}$$

(b) Check whether the transversatility condition holds.

Question 4

Suppose Ω is a bounded, open set in $\mathbb{R}^n$, $u\in C^1(\Omega)$ and $\varphi\in C_c^\infty(\Omega)$. Show that

$$\int_\Omega u\frac{\partial\varphi}{\partial x_i}\,\mathrm{d}x = -\int_\Omega \frac{\partial u}{\partial x_i}\varphi\,\mathrm{d}x\,.$$

(*Hint:* Note that we may not have $\partial\Omega$ is C^1 or $u\in C^1(\bar\Omega)$. But note that $u\varphi\in C_c^1(\Omega)$: so extend by zero to all of $\mathbb{R}^n$ and apply Gauss-Green to a ball containing Ω.)

Deadline for handing in: 0800 Wednesday 14 January

Please put solutions in Box 17, 1st floor (near the library)

Homepage: `http://www.mathematik.uni-muenchen.de/~soneji/pde1.php`