MATHEMATISCHES INSTITUT
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PDG II
(Tutorium)

Tutorial 9

Exercise 1

This exercise concerns the proof of Theorem 2.26 from the lecture (see lecture for notation.) Recall that $u \in W^{1,p}(U) \cap C^\infty(U)$.

(a) Prove that $|u(x + he_i) - u(x)| \leq |h| \int_0^1 |Du(x + the_i)| dt$ implies

$$\int_V |D^h u|^p \, dx \leq C \int_U |Du|^p \, dx. \tag{1}$$

(b) Use approximation to prove that (1) holds for all $u \in W^{1,p}(U)$.

(c) Prove the "integration by parts"-formula for difference quotients:

$$\int_V u(D_i^h \varphi) \, dx = - \int_V (D_i^{-h} u)\varphi \, dx, \quad \varphi \in C_c^\infty(V). \tag{2}$$

Prove that (2) still holds for $\varphi \in H_0^1(V)$.

Exercise 2

This exercise concerns the proof of Theorem 2.27 in the lecture (for notation, see the lecture).

(a) Let $u \in H^1(U)$, $\zeta \in C_c^\infty(U)$. Prove that $v := -D_k^{-h}(\zeta^2 D_k^h u) \in H_0^1(U)$.

(b) Let $f^h(x) := f(x + he_k)$. Prove "Leibniz' rule" for difference quotients:

$$D_k^h(fg) = f^h D_k^h g + g D_k^h f.$$

(c) Prove that

$$D_k^h(f_{x_j}) = (D_k^h f)_{x_j}.$$

Exercise 3

Assume $a_ij \in C^1(U)$, $b_i, c \in L^\infty(U)$, $f \in L^2(U)$, and that $u \in H^1(U)$ is a weak solution of

$$Lu = f\,. \tag{3}$$

Prove that if additionally we know that $u \in H^2_{\text{loc}}(U)$, then u is a *strong* solution of (3), i.e.

$$Lu = f \quad \text{a.e. in } U\,.$$