

PDG II (Tutorium)

Tutorial 6

Exercise 1

Prove: $W_0^{1,p}(\mathbb{R}^n) = W^{1,p}(\mathbb{R}^n)$.

Exercise 2

Let U be a bounded, open subset of $\mathbb{R}^n$, and suppose ∂U is C^1 . Assume $n < p \leq \infty$, and $u \in W^{1,p}(U)$. Then u has a version $u^* \in C^{0,\gamma}(\overline{U})$, for $\gamma = 1 - n/p$, with the estimate

$$\|u^*\|_{C^{0,\gamma}(\overline{U})} \leq C \|u\|_{W^{1,p}(U)}.$$

The constant C depends only on p, n and U .

Exercise 3

Prove Theorem 1.29 in the Lecture (Rellich-Kondrachov): Let U be a bounded, open subset of $\mathbb{R}^n$, and suppose ∂U is C^1 . Assume $1 \leq p < n$. Then

$$W^{1,p}(U) \subset\subset L^q(U)$$

for each $q \in [1, p^*)$.