MATHEMATISCHES INSTITUT
DER UNIVERSITÄT MÜNCHEN
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PDG II
(Tutorium)

Tutorial 4

In the following, $U \subset \mathbb{R}^n$ will always denote an open set.

Exercise 1

Assume U is bounded. Prove that for all $p \in [1, \infty]$ and all $q \in [1, p]$:

$$\|u\|_{L^q(U)} \leq C\|u\|_{L^p(U)}$$

and determine $C = C(q, p, U)$.

Exercise 2 Do some of the missing details in the proof of Theorem 1.14 (trace theorem) in the Lecture:

(a) Prove Young's inequality:
Let $1 < p, q < \infty$, $\frac{1}{p} + \frac{1}{q} = 1$. Then $ab \leq \frac{a^p}{p} + \frac{b^q}{q}$ for all $a, b \geq 0$.

(b) Use integration by parts (Gauss-Green theorem) to prove (for notation, see the Lecture):

$$\int_{\{x_n=0\}} \zeta |u|^p \, dx' = -\int_{B^+} \left(\zeta |u|^p\right)_{x_n} dx \, .$$

(c) Prove the inequality (for notation, see the Lecture):

$$-\int_{B^+} \left(|u|^p \zeta_{x_n} + p|u|^{p-1}(\operatorname{sgn} u) u_{x_n} \zeta\right) dx \leq C \int_{B^+} \left(|u|^p + |Du|^p\right) dx \, .$$

Exercise 3

Prove the *General Hölder inequality*: let $1 \leq p_1, \ldots, p_m \leq \infty$, with $\frac{1}{p_1} + \frac{1}{p_2} + \cdots + \frac{1}{p_m} = 1$, and assume $u_k \in L^{p_k}(U)$ for $k = 1, \ldots, m$. Then

$$\int_U |u_1 \cdots u_m| \, dx \leq \prod_{k=1}^{m} \|u_k\|_{L^{p_k}(U)} \, .$$