# PDG II
# (Tutorium)

## Tutorial 12

### Exercise 1

This exercise concerns the proof of Theorem 2.42 from the lecture (see lecture for notation.)

(a) Prove that $B[u, u] = \sum_{k=1}^{\infty} d_k^2 \lambda_k$.

(b) Prove that

$$\min \left\{ B[u, u] : u \in H_0^1(U),\ \|u\|_{L^2(U)} = 1 \right\} = \min_{\substack{u \in H_0^1(U) \\ u \neq 0}} \frac{B[u, u]}{\|u\|_{L^2(U)}^2}.$$

(c) Prove that $\int_U u^+ u^- dx = 0$ and $B[u^+, u^-] = B[u^-, u^+] = 0$.

(d) Prove that $B[\cdot, \cdot]$ defines a scalar product on $H_0^1(U)$.

(e) Prove that the resulting norm is equivalent to the $H_0^1(U)$-norm.

(f) Prove that if $u = \sum_{k=1}^{m} (u, w_k) w_k$ and $w_k$, $k = 1, \ldots, m$, solve $L w_k = \lambda_1 w_k$ weakly, then $u$ solves $Lu = \lambda_1 u$ weakly.

### Exercise 2

This exercise concerns the proof of Theorem 2.39 from the lecture (see lecture for notation.)

(a) Prove that $\ker(S) = \{0\}$.

(b) Prove that all eigenvalues of $S$ are positive.

(c) Why is $\lambda_1 > 0$?

### Exercise 3

Remaining non-discussed questions from all previous tutorial sheets.