The Rellich inequality in $L^p(\Omega)$

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The talk will describe joint work with David Edmunds on the inequality

$$\int_\Omega \frac{|u(x)|^p}{\delta^{2p}} dx \leq K(p,n) \int_\Omega |\Delta u(x)|^p dx, \quad u \in C_0^\infty(\Omega),$$

where Ω is a non-empty, proper open subset of $\mathbb{R}^n$, $\delta(x)$ is the distance from $x \in \Omega$ to the boundary of Ω and $p \in (1,\infty)$. If Ω is convex and $p = 2$ then $K(2,n) \leq 16/9$, but for $p \neq 2$ more effort is required to obtain a good estimate for $K(n,p)$. The work reported will use results derived by Evans and Lewis, and a remarkable result of Bañuelos /Wang and Iwaniec/Martin which gives a precise value for the norms of Riesz transforms.