Mathematisches Institut
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Wintersemester 2006/7
Blatt 9

Übungen zur Vorlesung "Recursion Theory"

Aufgabe 33. Spector's theorem says that if $a, b \in \mathcal{O}$ and $|a|_{\mathcal{O}} \leq |b|_{\mathcal{O}}$, then H_a is recursive in H_b, effectively in a and b. It is proved by applying the Recursion Lemma to M and $\prec$, w.r.t. the relation R:

$$M := \{ \langle a, b \rangle \mid a \in \mathcal{O} \wedge b \in \mathcal{O} \wedge |a|_{\mathcal{O}} \leq |b|_{\mathcal{O}} \}$$

$$\langle a, b \rangle \prec \langle c, d \rangle \;:\leftrightarrow\; (a <_{\mathcal{O}} c \wedge b \leq_{\mathcal{O}} d) \vee (a \leq_{\mathcal{O}} c \wedge b <_{\mathcal{O}} d),$$

$$R(p, e) \;:\leftrightarrow\; p \in M \wedge H_{(p)_0} \leq [e]\, H_{(p)_1}.$$

Fix $a, b \in \mathcal{O}$ such that $|a|_{\mathcal{O}} \leq |b|_{\mathcal{O}}$ and assume

$$\forall_{c,d} \left(\langle c, d \rangle \prec \langle a, b \rangle \to H_c \leq [\varphi_t \langle c, d \rangle]\, H_d \right). \tag{1}$$

We need to define a recursive function χ such that $H_a \leq [\chi(t, a, b)]\, H_b$. Do the proof in the case where a denotes a limit.

Aufgabe 34. If a set A is Δ^1_1-definable in a Δ^1_n-definable set B, then A is Δ^1_n-definable.

Aufgabe 35. To prove that for every $a \in \mathcal{O}$ the set H_a is Δ^1_1-definable effectively in a (i.e., that there is a recursive function f such that for every $a \in \mathcal{O}$ the number $(f(a))_0$ is a Π^1_1-index of H_a and $(f(a))_1$ is a Π^1_1-index of $\mathbb{N} \setminus H_a$), one applies the Recursion Lemma to $\mathcal{O}$ and $<_{\mathcal{O}}$ with respect to

$$R(a, e) \;:\leftrightarrow\; a \in \mathcal{O} \wedge (e)_0 \text{ is } \Pi^1_1\text{-index of } H_a \wedge (e)_1 \text{ is } \Pi^1_1\text{-index of } \mathbb{N} \setminus H_a.$$

So fix $a \in \mathcal{O}$ and assume $\forall_{b <_{\mathcal{O}} a} R(b, \varphi_t(b))$. We need to define a recursive function χ such that $R(a, \chi(t, a))$. Do the proof when a denotes a limit.

Aufgabe 36. Prove that any two disjoint Σ^1_1-definable sets A and B can be separated by a hyperarithmetical set C.
Hint. Because of the Π^1_1-completeness of $\mathcal{O}$ there is a recursive function f such that $n \notin B \;\leftrightarrow\; f(n) \in \mathcal{O}$.

Abgabetermin. Donnerstag, 25. Januar 2007, 11:15 Uhr