

Übungen zur Vorlesung “Recursion Theory”

Aufgabe 29. Let Γ be a monotone operator. Define $\Gamma \downarrow \alpha$ by transfinite recursion on all ordinals α :

$$\begin{aligned}\Gamma \downarrow 0 &:= U, \\ \Gamma \downarrow (\alpha + 1) &:= \Gamma(\Gamma \downarrow \alpha), \\ \Gamma \downarrow \lambda &:= \bigcap_{\xi < \lambda} \Gamma \downarrow \xi.\end{aligned}$$

Prove for all ordinals α :

- (a) $\Gamma \downarrow \alpha \supseteq \Gamma \downarrow (\alpha + 1)$.
- (b) $\Gamma \downarrow \alpha \supseteq \mathcal{C}_\Gamma$.

Aufgabe 30. Let Γ be a monotone operator. Prove for all ordinals α :

$$\Gamma \downarrow \alpha = \Gamma \downarrow (\alpha + 1) \rightarrow \mathcal{C}_\Gamma = \Gamma \downarrow \alpha.$$

Aufgabe 31. Let Γ be a monotone operator. Show that there exists an ordinal β such that

$$\Gamma \downarrow \beta = \mathcal{C}_\Gamma.$$

Aufgabe 32. Prove that $a \not\prec_{\mathcal{O}} a$, for all $a \in \mathbb{N}$. (*Hint.* Induction on $|a|_{\mathcal{O}}$, using transitivity of $<_{\mathcal{O}}$).

Abgabetermin. Donnerstag, 21. Dezember 2006, 11:15 Uhr