Mathematisches Institut
der Universität München
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Wintersemester 2006/7
Blatt 7

Übungen zur Vorlesung "Recursion Theory"

Aufgabe 25. Show that the set $\mathcal{D}_\infty$ of all unary functions whose graph is in Δ^0_∞ is analytical, and estimate its position in the analytical hierarchy. *Hint.* Recall the definition of the universal Σ^0_{r+1}-definable relations $U^0_{r+1}(e,\vec{n})$:

$$
\begin{aligned}
U^0_1(e,\vec{n}) &:\leftrightarrow \exists_s T(e,\vec{n},s) \quad (\leftrightarrow\ \vec{n} \in W^{(q)}_e),\\
U^0_{r+1}(e,\vec{n}) &:\leftrightarrow \exists_m \neg U^0_r(e,\vec{n},m).
\end{aligned}
$$

Aufgabe 26. Let U be a non-empty set, and $\Gamma\colon \mathcal{P}(U) \to \mathcal{P}(U)$ a monotone operator.

$$\mathcal{C}_\Gamma := \bigcup \{\, X \subseteq U \mid X \subseteq \Gamma(X) \,\}.$$

is called the set defined *co-inductively* by Γ. Prove that

(a) If $X \subseteq \Gamma(X)$, then $X \subseteq \mathcal{C}_\Gamma$.
(b) $\Gamma(\mathcal{C}_\Gamma) = \mathcal{C}_\Gamma$.

Hence $\mathcal{C}_\Gamma$ is the greatest fixed point of Γ.

Aufgabe 27. Let U be a non-empty set, and $\prec$ a binary relation on U. By a *reduction sequence* we mean a finite or infinite sequence $u_1, u_2, \dots$ of elements of U such that $u_i \succ u_{i+1}$ ($i \geq 1$). Show that $u \in \mathrm{acc}_\prec$ if and only if every reduction sequence starting with u terminates after finitely many steps.

Aufgabe 28. Show that every Σ^0_1-definable monotone operator Γ is continuous.

Abgabetermin. Montag, 11. Dezember 2006, 11:15 Uhr