Mathematisches Institut
der Universität München
Helmut Schwichtenberg

Wintersemester 2006/7
Blatt 5

Übungen zur Vorlesung "Recursion Theory"

Aufgabe 17. (a) Prove that the elementary functions (like the primitive recursive ones) form an "honest" class in the sense that every elementary function is computable within a number of steps bounded by some (other) elementary function.

(b) Prove that the elementary functions are just those definable by L_2-programs, since

$$\mathrm{Elem} = \bigcup_i \mathrm{Comp}(F_2^i)$$

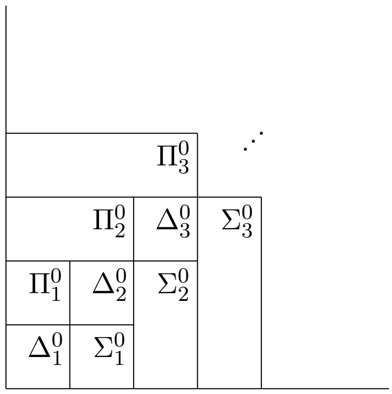
where $F_2(n) = n \cdot 2^n$.

Aufgabe 18. (a) Prove that the Δ_3^0-set

$$A := \{\, 2\langle e, n\rangle \mid \exists_m \forall_s \neg T(e,n,m,s) \,\} \cup \{\, 2\langle e,n\rangle + 1 \mid \forall_m \exists_s T(e,n,m,s) \,\},$$

is not a Σ_2^0-set.

(b) Prove that all inclusions in the figure below are proper.



Aufgabe 19. In the course the following lemma was proved: For all partial recursive functionals Φ, $\Psi_1, \dots, \Psi_p$ we can find a partial recursive functional Φ' such that for all $\vec{g}, \vec{n}$ with $\lambda x \Psi_i(\vec{g}, \vec{n}, x)$ total we have

$$\Phi'(\vec{g}, \vec{n}) \;=\; \Phi(\lambda x \Psi_1(\vec{g}, \vec{n}, x), \dots, \lambda x \Psi_p(\vec{g}, \vec{n}, x), \vec{n}).$$

Show that the lemma is false without the restriction on the arguments $\vec{g}, \vec{n}$.

Aufgabe 20. A set B is said to be reducible to a set A if there is a total recursive function f such that for all n

$$n \in B \;\leftrightarrow\; f(n) \in A.$$

Therefore if B is reducible to A, then B is recursive in A. Show that the converse is false.

Abgabetermin. Donnerstag, 23. November 2006, 11:15 Uhr