Mathematisches Institut
der Universität München
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Wintersemester 2006/7
Blatt 4

Übungen zur Vorlesung "Recursion Theory"

Aufgabe 13. Prove the following:

(a) For every total computable function $f\colon \mathbb{N} \to \mathbb{N}$ there exists a total computable function $g\colon \mathbb{N}^2 \to \mathbb{N}$, such that

$$\forall_{n \in \mathbb{N}}(f(n) = \mu_m(g(n,m) = 0)).$$

(b) There exists a computable function $\psi\colon \mathbb{N} \to \mathbb{N}$ such that there is no total computable function $g\colon \mathbb{N}^2 \to \mathbb{N}$ for which

$$\forall_{n \in \mathbb{N}}(\psi(n) = \mu_m(g(n,m) = 0)).$$

Aufgabe 14. (a) Show that for every term $t(\vec{\psi}; \vec{n})$ (in the sense of Sec.1.4.1) one of the following two alternatives holds:

(i) For all terms $\vec{s}$ the term $t(\vec{\psi}, \vec{s})$ has a defined value (Example: $\mathrm{dc}(0,0,0,n)$).

(ii) There is an index i such that for all terms $\vec{s}$ the term $t(\vec{\psi}, \vec{s})$ has a defined valu only if s_i has (Example: $\mathrm{dc}(n_1,n_2,n_3,n_4)$, $i = 1$ or $i = 2$). (Sequentiality)

(b) Conclude that there is no term $t(\vec{\psi}; n_1, n_2, n_3, n_4)$ such that

$$t(\vec{\psi}; s_1, s_2, s_3, s_4) = \begin{cases} s_3 & \text{if } s_1, s_2 \text{ are both defined and equal} \\ s_4 & \text{if } s_1, s_2 \text{ are both defined and unequal} \\ s_3 & \text{if } s_3, s_4 \text{ are both defined and equal} \\ \text{undefined} & \text{otherwise.} \end{cases}$$

Aufgabe 15. The sequence $F_0, F_1, \dots F_k, \dots$ of Prim functions is given by $F_0(n) \;=\; n+1$, $F_{k+1}(n) \;=\; F_k^n(n)$. Show that

(a) L_k-COMPUTABLE $= \bigcup_i \mathrm{Comp}(F_k^i)$, for each $k \geq 1$.

(b) Conclude that $\mathrm{Prim} = \bigcup_k \mathrm{Comp}(F_k)$.

Aufgabe 16. Show that the "Ackermann-Péter Function" $F\colon \mathbb{N}^2 \to \mathbb{N}$ defined as

$$F(k,n) = F_k(n)$$

is not primitive recursive.

Abgabetermin. Donnerstag, 16. November 2006, 11:15 Uhr