Mathematisches Institut
der Universität München
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Wintersemester 2006/7
Blatt 3

**Übungen zur Vorlesung "Recursion Theory"**

**Aufgabe 9.** (a) Let

$$R_i(\vec{n}) \leftrightarrow \exists_{k_1} \dots \exists_{k_{l_i}} E_i(\vec{n}, k_1, \dots, k_{l_i}) \quad (i = 1, 2)$$
$$R(\vec{n}) \leftrightarrow R_1(\vec{n}) \wedge R_2(\vec{n})$$

with $E_i$ elementary. Show that $R$ is $\Sigma^0_1$-definable.

(b) Let

$$R(\vec{n}, k) \leftrightarrow \exists_{k_1} \dots \exists_{k_l} E(\vec{n}, k, k_1, \dots, k_l)$$
$$S(\vec{n}, m) \leftrightarrow \forall_{k<m} R(\vec{n}, k)$$

with $E$ elementary. Show that $S$ is $\Sigma^0_1$-definable. (Hint. Use sequence coding).

**Aufgabe 10.** Prove that there is no total computable universal function for the class of $k$-ary total computable functions, i.e., there is no total computable function $f\colon \mathbb{N}^{k+1} \to \mathbb{N}$, such that

$$\forall_{g\colon \mathbb{N}\to\mathbb{N} \text{ total computable}} \exists_{e_f \in \mathbb{N}} \forall_{\vec{n} \in \mathbb{N}^k} (f(e_f, \vec{n}) = g(\vec{n})).$$

**Aufgabe 11.** Prove that the recursive program

$$(PR) \quad \begin{cases} q(n, m, 0) &= n, \\ q(n, m, i + 1) &= p(q(n, m, i), m \dot{-} (i + 1)), \end{cases}$$
$$(C) \quad g'(n, m) \ = \ g(q(n, m, m)),$$
$$(C) \quad h'(n, m, i, j) \ = \ h(q(n, m, m \dot{-} (i + 1)), i, j),$$
$$(PR) \quad \begin{cases} f'(n, m, 0) &= g'(n, m), \\ f'(n, m, i + 1) &= h'(n, m, i, f'(n, m, i)), \end{cases}$$
$$(C) \quad f(n, m) \ = \ f'(n, m, m).$$

defines the function

$$\begin{cases} f(n, 0) &= g(n), \\ f(n, m + 1) &= h(n, m, f(p(n, m), m)). \end{cases}$$

**Aufgabe 12.** Let

$$\mathcal{P} = \{\, p\colon \mathbb{N} \to \mathbb{N} \mid \exists_{n\in\mathbb{N}} \exists_{a_0,\dots,a_n\in\mathbb{N}} \forall_{x\in\mathbb{N}} \big(p(x) = \sum_{i=0}^{n} a_i x^i\big) \,\}$$

be the class of all polynomials of one variable with coefficients in $\mathbb{N}$. Prove that

(a) There is an elementary universal function $U$ for the class $\mathcal{P}$, such that

$$\forall_{p\in\mathcal{P}} \exists_{e_p\in\mathbb{N}} \forall_{x\in\mathbb{N}} (U(e_p, x) = p(x)).$$

(b) There is no universal function of $\mathcal{P}$, which itself is in $\mathcal{P}$.

**Abgabetermin.** Donnerstag, 9. November 2006, 11:15 Uhr