Mathematisches Institut
der Universität München
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Wintersemester 2006/7
Blatt 2

Übungen zur Vorlesung "Recursion Theory"

Aufgabe 5. For every sequence $a_0, \dots, a_{n-1} < b$ of numbers less than b we can find a number c such that $\beta(c,i) = a_i$ for all $i < n$. In the lecture course c was defined by

$$a := \pi(b,n), \qquad d := \prod_{i<n} \bigl(1 + \pi(a_i, i)a!\bigr), \qquad c := \pi(a!, d).$$

Show that $c \le 4 \cdot 4^{n(b+n+1)^4}$.

Aufgabe 6. The class $\mathcal{E}_1$ consists of those number theoretic functions which can be defined from the initial functions: constant 0, successor S and projections (onto the ith coordinate) by applications of composition and exponentially bounded recursion according to the scheme

$$\begin{aligned} f(\vec{m}, 0) &= g(\vec{m}), \\ f(\vec{m}, n+1) &= h(n, f(\vec{m}, n), \vec{m}), \\ f(\vec{m}, n) &\le 2_k(\max(\vec{m}, n)). \end{aligned}$$

Show that $\mathcal{E}_1$ is the class of elementary functions.

Aufgabe 7. Show that the class $\mathcal{E}$ is closed under limited course-of-values recursion. Thus if h, k are given functions in $\mathcal{E}$ and f is defined from them according to the scheme

$$\begin{aligned} f(\vec{m}, n) &= h(n, \langle f(\vec{m}, 0), \dots . f(\vec{m}, n-1)\rangle, \vec{m}) \\ f(\vec{m}, n) &\le k(\vec{m}, n) \end{aligned}$$

then f is in $\mathcal{E}$ also.

Aufgabe 8. Show that a relation R is computable if and only if both R and its complement $\mathbb{N}^n \setminus R$ are Σ^0_1-definable.

Abgabetermin. Donnerstag, 2. November 2006, 11:15 Uhr