Mathematisches Institut
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Wintersemester 2006/7
Blatt 10

Übungen zur Vorlesung "Recursion Theory"

Aufgabe 37. Prove that the set $\mathcal{H}$ of all hyperarithmetical functions is Π^1_1-, but not Σ^1_1-definable.

Aufgabe 38. Prove that the following are equivalent.
(a) $f \in \mathcal{H}$.
(b) G_f is Σ^1_1-definable.
(c) G_f is Π^1_1-definable.
Conclude that if P is a Π^1_1-definable relation, then

$$\forall_n \exists_m P(n,m) \to \exists_{f \in \mathcal{H}} \forall_n P(n, f(n)).$$

Aufgabe 39. Prove the following strengthening of the conclusion above: If P is a Π^1_1-definable relation, then

$$\forall_n \exists_{f \in \mathcal{H}} P(f,n) \to \exists_{g \in \mathcal{H}} \forall_n P((g)_n, n).$$

Here $(g)_n(m) := g(\langle n, m \rangle)$.

Aufgabe 40. Show that there is a (1,1)-ary arithmetical relation Q such that for every $a \in \mathcal{O}$ the $(1,0)$-ary relation $Q_{(a)}$ implicitely defines $\langle c_{J_a} \rangle$; here $Q_{(a)}(f) :\leftrightarrow Q(f,a)$.

Abgabetermin. Donnerstag, 2. Februar 2007, 11:15 Uhr