Mathematisches Institut
der Universität München
Helmut Schwichtenberg

Wintersemester 2006/7
Blatt 1

Übungen zur Vorlesung "Recursion Theory"

Aufgabe 1. Show that exponentiation defined by $x^0 := 1$, $x^{y+1} := x \cdot x^y$ is register machine computable.

Aufgabe 2. Prove the following.
(a) For every subelementary function $f \colon \mathbb{N}^r \to \mathbb{N}$ there is a number k such that for all $\vec{n} = n_1, \dots, n_r$,

$$f(\vec{n}) < \max(\vec{n}, 2)^k.$$

(b) Conclude that the function $n \mapsto 2^n$ is not subelementary.

Aufgabe 3. Show that the following functions and relations of number theory are elementary: $\lfloor \frac{m}{n} \rfloor$, $m \bmod n$, $\mathrm{Prime}(m)$, p_n, where $p_0, p_1, p_2, \dots$ gives the enumeration of primes in increasing order. (Hint: you may want to use the estimate $p_n \leq 2^{2^n}$ for the nth prime p_n, which can be proved by induction on n).

Aufgabe 4. Enumerate the pairs of natural numbers as follows:

$$\begin{array}{lllll}
\vdots \\
10 \\
6 & \dots \\
3 & 7 & \dots \\
1 & 4 & 8 & \dots \\
0 & 2 & 5 & 9 & \dots
\end{array}$$

Let $\pi(a, b)$ be the number written at position (a, b), so $\pi \colon \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ is a bijection. Let π_1, π_2 be the components of its inverse, so

$$\pi_1(\pi(a, b)) = a,$$
$$\pi_2(\pi(a, b)) = b,$$
$$\pi(\pi_1(c), \pi_2(c)) = c.$$

Show the following.
(a) π, π_1 and π_2 are subelementary.
(b) $\pi(a, b) < (a + b + 1)^2$,
(c) $\pi_1(c), \pi_2(c) \leq c$.

Abgabetermin. Donnerstag, 27. Oktober 2006, 11:15 Uhr