

Homework for the lecture course „Mathematical Logic“

The material in this homework sheet is from F. Wiesnet, Verified Program Extraction in Number Theory, to appear 2026 in Annals of Pure and Applied Logic (APAL).

Problem 49. (6 points). Prove informally that $\gcd(n, m)$ can be linearly combined from n, m , i.e.,

$$\exists_{l_0, l_1} (\gcd(n, m) + l_0 n = l_1 m \vee \gcd(n, m) + l_0 m = l_1 n)$$

Hint. Prove by induction on l

$$\forall_{l, n, m} (n < m < l \rightarrow \exists_{l_0, l_1} (\gcd(n, m) + l_0 n = l_1 m \vee \gcd(n, m) + l_0 m = l_1 n))$$

In this proof compare $m - n$ with $n + 1$. In the \neq -cases use the IH to find out which part of the disjunction could be proved, and of l_0, l_1 .

Problem 50. (4 points). Prove in Minlog the formula from Problem 49 from the auxiliary proposition in the hint (cf. ueb13.scm)

Problem 51. (6 points). Prove in Minlog the auxiliary proposition in Problem 49 (cf. ueb13.scm)

Due. Wednesday, 28. January 2026, 8:00.