

**Homework for the lecture course „Mathematical Logic“**

The material in this homework sheet is from F. Wiesnet, Verified Program Extraction in Number Theory, to appear 2026 in Annals of Pure and Applied Logic (APAL).

**Problem 49.** (6 points). Prove informally that  $\gcd(n, m)$  can be linearly combined from  $n, m$ , i.e.,

$$\exists_{l_0, l_1} (\gcd(n, m) + l_0n = l_1m \vee \gcd(n, m) + l_0m = l_1n)$$

Hint. Prove by induction on  $l$

$$\forall_{l, n, m} (n < m < l \rightarrow \exists_{l_0, l_1} (\gcd(n, m) + l_0n = l_1m \vee \gcd(n, m) + l_0m = l_1n))$$

In this proof compare  $m - n$  with  $n + 1$ . In the  $\neq$ -cases use the IH to find out which part of the disjunction could be proved, and of  $l_0, l_1$ .

**Problem 50.** (4 points). Prove in Minlog the formula from Problem 49 from the auxiliary proposition in the hint (cf. ueb13.scm)

**Problem 51.** (6 points). Prove in Minlog the auxiliary proposition in Problem 49 (cf. ueb13.scm)

**Due.** Wednesday, 28. January 2026, 8:00.