

Homework for the lecture course „Mathematical Logic“

Problem 37. (4 points). Prove informally that for all natural numbers m, n there are natural numbers q, r such that

$$n = (m + 1)q + r \quad \text{and} \quad r \leq m.$$

Problem 38. (4 points). Prove informally that for all natural numbers a, n_0, \dots, n_{k-1} with $a = \langle n_0, \dots, n_{k-1} \rangle$ we have

- (a) $\text{lh}(a) = k$,
- (b) $(a)_i = n_i$ for all $i < k$.

Problem 39. (4 points). Prove informally that the graph of the function $n \mapsto 2_n(1)$ is elementary. Hint. Coding of finite lists, and closure of elementary relations under bounded quantifiers.

Problem 40. (4 points). Formalize the proof from Problem 37 in Minlog (see `ueb10.scm`).

Due. Wednesday, 7. January 2026, 8:00.