Mathematisches Institut der LMU
Helmut Schwichtenberg

Winter semester 2025/26
hwk06

Homework for the lecture course „Mathematical Logic“

Problem 21. (4 points). Give derivations for
(a) $\vdash_c ((A \to B) \to A) \to A$ (Peirce formula).
(b) $\vdash ((((A \to B) \to A) \to A) \to B) \to B$ (Mints formula).

Problem 22. (4 points). Prove $\nvdash_i ((A \to B) \to A) \to A$. Hint. Use the tree model given in the course to prove $\nvdash_i \neg\neg P \to P$.

Problem 23. (4 points). Let $\mathcal{T}$ be a tree model, $t, r(x)$ terms, $A(x)$ a formula and η an assignment in $|\mathcal{T}|$. Prove
(a) $\eta(r(t)) = \eta_x^{\eta(t)}(r(x))$.
(b) $\mathcal{T}, k \Vdash A(t)[\eta]$ iff $\mathcal{T}, k \Vdash A(x)[\eta_x^{\eta(t)}]$.
Hint: Induction on terms and formulas.

Problem 24. (4 points).
(i) Formalize the derivations from Problem 21 by hand (i.e., in single steps) in Minlog. Compute the corresponding derivation terms by evaluating

(`proof-to-expr-with-formulas`)

after finishing the proof (see `ueb06.scm`).
(ii) Generate the derivations from Problem 21 automatically, i.e., by applications of `(prop)`.
(iii) What is the normal form of the derivation of the Mints formula found by `(prop)`?

Due. Wednesday, 26. November 2025, 8:00.