Mathematisches Institut der LMU
Helmut Schwichtenberg

Winter semester 2025/26
hwk05

Homework for the lecture course „Mathematical Logic“

Problem 17. (4 points). Give derivations for
(a) $(\bot \to B) \to (A \to B \tilde{\vee} C) \to (A \to B) \tilde{\vee} (A \to C)$,
(b) $(A \to B) \tilde{\vee} (A \to C) \to A \to B \tilde{\vee} C$.

Problem 18. (4 points). Prove that for all formulas A without $\vee$, $\exists$ we have
(a) $\vdash_c A \to A^g$,
(b) $\vdash_c A^g \to A$.
Hint. Prove (a) and (b) by (simultaneous) induction on A.

Problem 19. (4 points). Let $\mathcal{T} = (D, I_0, I_1)$ be a tree model on a finitely branching tree T. With k, k' we denote nodes, i.e., finite sequences $\langle a_0, a_1, \dots, a_{n-1} \rangle$ of elements of a given finite set S. We write $k \preceq k'$ if k is an initial seqment of k'. Let η be an assignment in D, i.e., a map assigning to each variable $x \in \mathrm{dom}(\eta)$ a value $\eta(x) \in D$. Prove that for all formulas A

$$k \Vdash A[\eta] \text{ and } k \preceq k' \text{ implies } k' \Vdash A[\eta].$$

Problem 20. (4 points). Formalize the derivations from Problem 17 in Minlog. Compute the corresponding derivation term by evaluating

(`proof−to−expr−with−formulas`)

after finishing the proof (see `ueb05.scm`).

Due. Wednesday, 19. November 2025, 8:00.