Mathematisches Institut der LMU
Helmut Schwichtenberg

Winter semester 2025/26
hwk04

Homework for the lecture course „Mathematical Logic“

Problem 13. (4 points). Give derivations for

(a) $(A \mathbin{\tilde{\vee}} B \to C) \to A \to C$,
(b) $(A \mathbin{\tilde{\vee}} B \to C) \to B \to C$,
(c) $(\neg\neg C \to C) \to (A \to C) \mathbin{\tilde{\vee}} (B \to C) \to A \to B \to C$.

Problem 14. (2 points). A derivation for $(\neg\neg B \to B) \to \neg\neg(A \to B) \to A \to B$ is

$$\dfrac{u\colon \neg\neg B \to B \qquad \dfrac{\dfrac{v\colon \neg\neg(A \to B) \qquad \dfrac{\dfrac{u_1\colon \neg B \qquad \dfrac{u_2\colon A \to B \qquad w\colon A}{B}}{\bot}}{\neg(A \to B)}\,{\to^+}u_2}{\bot}}{\neg\neg B}\,{\to^+}u_1}{B}$$

where applications of $\to^+$ at the end are omitted. What is the corresponding derivation term?

Problem 15. (6 points). The relation $\succ_1$ is called *confluent* (or Church-Rosser, CR) if $M \succ_1 M'$ and $M \succ_1 M''$ together imply the existence of an N such that $M' \succ_1 N$ and $M'' \succ_1 N$. We call $\succ_1$ *weakly confluent* (or weak Church-Rosser, WCR) if $M \succ_1 M'$ and $M \succ_1 M''$ together imply the existence of some N such that $M' \succeq N$ and $M'' \succeq N$. Prove

(a) the normal form of a strongly normalizing M is uniquely determined.
(b) If all M are strongly normalizing, then $\succeq$ is confluent.

Hint. M is called *good* if $M \succ_1 K$ and $M \succ_1 L$ together imply the existence of an N such that $K \succ_1 N$ and $L \succ_1 N$. Prove that every strongly normalizing M is good. Use transfinite induction on the well-founded partial ordering $\prec$, i.e., the scheme

$$\forall_{M \in \mathcal{T}}(\forall_{M' \prec M} P(M') \to P(M)) \to \forall_{M \in \mathcal{T}} P(M).$$

Problem 16. (4 points).

(a) Formalize the derivations from Problem 13 in Minlog.
(b) Formalize the derivation from Problem 14 in Minlog. Compute the corresponding derivation term by evaluating

    (`proof-to-expr-with-formulas`)

    after finishing the proof.

(see `ueb04.scm`).

Due. Wednesday, 12. November 2025, 8:00