Prof. Dr. Bachmann
A. Dietlein, R. Schulte

PARTIAL DIFFERENTIAL EQUATIONS I
TUTORIAL SHEET 10

WS 2016/17
January 9, 2017

T 1. Let $d \in \mathbb{R}$. Prove that if $u, v \in C^2((0,\infty) \times \mathbb{R})$ are solutions of

$$\begin{cases} u_t + u_x = d(v-u) \\ v_t - v_x = d(u-v). \end{cases}$$

then they also solve the *Telegraph Equation*: $w_{tt} + 2dw_t - w_{xx} = 0$.

T 2. Let $g, h : [0,\infty) \to \mathbb{R}$. Find a solution of the boundary value problem

$$\begin{cases} u_{tt} - u_{xx} = 0 & \text{in } (0,\infty)^2 \\ u(0,\cdot) = g, u_t(0,\cdot) = h & \text{on } (0,\infty) \\ u_x(\cdot, 0) = 0 & \text{on } (0,\infty) \end{cases}$$

under the assumption that the even extensions $\widetilde{g}$ and $\widetilde{h}$ of g and h satisfy $\widetilde{g} \in C^2(\mathbb{R})$ and $\widetilde{h} \in C^1(\mathbb{R})$.
Hint: Consider the initial value problem of the wave equation with initial data given by $\widetilde{g}$ and $\widetilde{h}$.

T 3. Let $f \in C_c^0(\mathbb{R})$ be a compactly supported function. Find a solution of the initial value problem

$$\begin{cases} u_{tt}(t,x) - u_{xx}(t,x) = f(x)\cos(t) & \text{for } (t,x) \in (0,\infty) \times \mathbb{R} \\ u(0,x) = 0, u_t(0,x) = 0 & \text{for } x \in \mathbb{R} \end{cases}$$

without referring to Duhamel's principle but via separation of variables (i.e. make the ansatz $u_p(t,x) = T(t)X(x)$ for a particular solution of the above wave equation - the solution will depend on a solution $v \in C^2(\mathbb{R})$ of the ordinary differential equation $v'' + v = f$).