Prof. Dr. Bachmann | PARTIAL DIFFERENTIAL EQUATIONS I | WS 2016/17
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T 1. (i) Assume that $E = (E^1, E^2, E^3), B = (B^1, B^2, B^2)$, with $E^i, B^i \in C^2((0,\infty) \times \mathbb{R}^3)$ for $i = 1, 2, 3$, solve *Maxwell's equations*:

$$E_t = \nabla \times B, \qquad B_t = -\nabla \times E,$$
$$\nabla \cdot B = 0, \qquad \nabla \cdot E = 0.$$

Show that for $u = E^i$ or $u = B^i$, $i = 1, 2, 3$,

$$u_{tt} - \Delta u = 0.$$

Here, the curl *of a vector field* $F : \mathbb{R}^3 \to \mathbb{R}^3$ *is defined as*

$$\nabla \times F := (\partial_{x_2} F^3 - \partial_{x_3} F^2, \partial_{x_3} F^1 - \partial_{x_1} F^3, \partial_{x_1} F^2 - \partial_{x_2} F^1).$$

It satisfies the relation $\nabla \times (\nabla \times F) = \nabla(\nabla \cdot F) - \Delta F$.

(ii) Assume that $u = (u^1, u^2, u^3)$ with $u^i \in C^\infty((0,\infty) \times \mathbb{R}^3)$ solves the evolution equations of linear elasticity:

$$u_{tt} - \mu \Delta u - (\lambda + \mu)\nabla(\nabla \cdot u) = 0 \quad \text{in } (0,\infty) \times \mathbb{R}^3.$$

Show $v := \nabla \cdot u$ and $w := \nabla \times u$ each solve wave equations, but with different speeds of propagation.

T 2. Find all solutions $u \in C^2(\mathbb{R}^2)$ of the equation

$$u_{xy}(x, y) = 0.$$