Generalities:

Gauss-Green's theorem: $\int_\Omega u_{x_i}(x)\,\mathrm{d}x = \int_{\partial\Omega} u(x) n_i(x)\,\mathrm{d}S(x)$
Integration by parts: $\int_\Omega u_{x_i}(x)v(x)\,\mathrm{d}x = -\int_\Omega u(x)v_{x_i}(x)\,\mathrm{d}x + \int_{\partial\Omega} u(x)v(x)n_i(x)\,\mathrm{d}S(x)$
Green's formula (i): $\int_\Omega \Delta u(x)\,\mathrm{d}x = \int_{\partial\Omega} \frac{\partial u}{\partial n}(x)\,\mathrm{d}S(x)$
Green's formula (ii): $\int_\Omega \nabla u(x)\cdot\nabla v(x)\,\mathrm{d}x = -\int_\Omega u(x)\Delta v(x)\,\mathrm{d}x + \int_{\partial\Omega} u(x)\frac{\partial v}{\partial n}(x)\,\mathrm{d}S(x)$
Co-area formula for the ball: $\int_{B_r(x)} u(y)\,\mathrm{d}y = \int_0^r \left(\int_{\partial B_s(x)} u(y)\,\mathrm{d}S(y)\right)\mathrm{d}s$

Method of Characteristics:

First order PDE $F(x, u, \nabla u) = 0$ (initial values given at Σ) with characteristic equations ($z(t) := u(x(t))$, $p(t) := \nabla u(x(t))$) and non-characteristic condition: $\nabla_p F(\sigma, \eta, \pi)\cdot\nu(\sigma) \neq 0$ ($\nu(\sigma)$ normal vector to Σ at σ)

$$\begin{cases} \dot{x}(t) = \nabla_p F(x(t), z(t), p(t)) \\ \dot{z}(t) = p(t)\cdot\nabla_p F(x(t), z(t), p(t)) \\ \dot{p}(t) = -\nabla_x F(x(t), z(t), p(t)) - p(t)F_z(x(t), z(t), p(t)) \end{cases}$$

Laplace equation:

Laplace operator: $\Delta = \sum_{i\leq n} \partial_i^2$
Fundamental Solution:

$$\Phi(x,y) := \begin{cases} \frac{1}{(n-2)\omega_n}|x-y|^{2-n} & (n \geq 3) \\ -\frac{1}{2\pi}\ln|x-y| & (n = 2) \end{cases}$$

Poisson-kernel: $K_{B_r(x_0)}(x,y) := \frac{|r|^2-|x-x_0|^2}{r\omega_n|x-y|^n}$
Mean-Value Property: $u(x) = \frac{1}{r^{n-1}\omega_n}\int_{\partial B_r(x)} u(y)\,\mathrm{d}S(y)$
Superharmonic: $-\Delta u \geq 0$ or $H_B u \leq u$ with $H_B u(x) = \int_{\partial B} K_B(x,y)u(y)\,\mathrm{d}y$ for $x \in B$ and $H_B u(x) = u(x)$ otherwise.
Rotationally invariant functions : "$u(x) = f(|x|)$",
then $\Delta u(x) = f''(|x|) + \frac{n-1}{|x|}f'(|x|)$
Green's function: $G : \overline{\Omega}\times\Omega \to \mathbb{R}$ Green's function for Ω iff for all y:
(i) $x \to G(x,y) - \Phi(x,y)$ continuous on $\overline{\Omega}$, harmonic on Ω (ii) $x \to G(x,y) = 0$ on $\partial\Omega$.

Heat equation:

Heat operator: $\partial_t - \Delta$ (with $\Delta = \Delta_x$)
Fundamental Solution:

$$\Phi(t,x) := \begin{cases} \frac{1}{(4\pi t)^{n/2}}\exp(-\frac{|x|^2}{4t}) & (t,x) \in (0,\infty)\times\mathbb{R}^n \\ 0 & (t,x) \in \{0\}\times\mathbb{R}^n \end{cases}$$

Heat ball: $E(t,x;r) := \{(s,y) \in \mathbb{R}^{n+1} : s \leq t,\ \Phi(t-s, x-y) \geq r^{-n}\}$
Mean-Value Property: $u(t,x) = \frac{1}{r^n}\int_{E(t,x;r)} u(s,y)\frac{|x-y|^2}{4(t-s)^2}\,\mathrm{d}s\,\mathrm{d}y$

Wave equation:

Wave operator: $\partial_{tt} - \Delta$ (with $\Delta = \Delta_x$)
d'Alembert's formula (d=1): $u(t,x) = \frac{1}{2}(g(x+t) + g(x-t)) + \frac{1}{2}\int_{x-t}^{x+t} h(y)\,\mathrm{d}y$
Poisson's formula (d=2): $u(t,x) = \frac{2}{|B_t(x)|}\int_{B_t(x)} \frac{tg(z)+t\nabla g(z)\cdot(z-x)+t^2h(z)}{\sqrt{t^2-|z-x|^2}}\,\mathrm{d}z$
Kirchhoff's formula (d=3): $u(t,x) = \frac{1}{|\partial B_t(x)|}\int_{\partial B_t(x)} (g(y) + \nabla g(y)\cdot(y-x) + th(y))\,\mathrm{d}S(y)$
Energy Method: total energy $E(t) := K(t) + V(t)$ with potential energy
$V(t) := \frac{1}{2}\int_\Omega |\nabla u(t,x)|^2\,\mathrm{d}x$ and kinetic energy $K(t) := \frac{1}{2}\int_\Omega |u_t(t,x)|^2\,\mathrm{d}x$