

Reading course on Abelian varieties

In this reading course, the participants learn some basic results about abelian varieties, following mostly D. Mumford's book [3]. We meet each Monday 4-6 in B251. As a preparation for the meeting, the participants are required to read each week a certain portion of the literature, described below. Additionally, each week there will be one speaker who prepared a talk on the material that had to be read. A brief description on the talks is included below. Since every participant has to read the material of each talk, the talks are in general slightly shorter than usual seminar talks.

If not mentioned otherwise, all references in the program below refer to [3].

Prerequisites: Basic knowledge of algebraic geometry, e.g. [1, Chapter II and III].

1. Complex tori I, 22.10.2018

Speaker: T. Ose

Assigned reading: [3, §1, p. 1-4]

Talk: Present the material of [3, p. 1-4]. End by stating the theorem which computes $H^q(X, \Omega_X^p)$. If necessary, you can use as additional reference [2, I.2].

2. Complex tori II, 29.10.2018

Speaker: M. Paulsen

Assigned reading: [3, §1, p. 4-12]

Talk: Present the material of [3, p. 4-12]. Compute in particular $H^q(X, \mathcal{O}_X)$.

3. Abelian varieties: Definitions and basic results, 12.11.2018

Assigned reading: [3, §4]

Speaker: O. Hoyer

Talk: Define abelian varieties X . Present (i) " X is nonsingular", (iii) " Ω_X^1 is trivial" and (iv) " n multiplication by n is surjective if $\text{char}(k)$ does not divide n ". (Note that you should skip (ii)!) State and prove the rigidity lemma and prove Corollary 1, 2 and 3. Depending on the interest of the speaker (and on the time left), you may or may not address the content of Appendix to §4.

4. Rational maps into abelian varieties, 19.11.2018

Assigned reading: [2, I.3]

Speaker: C. Gürrer

Talk: Present the main results of [2, I.3].

5. Cohomology and base change I, 26.11.2018

Assigned reading: [1, III.8] and [3, §5, until Corollary 1]

Speaker: C. Stadlmayr

Talk: Recall the definition and basic properties of the higher direct image sheaves $R^p f_* \mathcal{F}$, where $f : X \rightarrow Y$ is a morphism and \mathcal{F} is a sheaf on X , see [1, III.8]. State and prove the main theorem of [3, §5].

6. **Cohomology and base change II, 03.12.2018**
Assigned reading: [3, §5, starting with Corollary 1]
Speaker: L. Lehmair
Talk: Recall the main theorem from last time. State and prove Corollaries 1, 2, 3, 4, 5 and 6.
7. **The theorem of the cube: I, 10.12.2018**
Assigned reading: [3, §6, p. 52–55]
Speaker: J. Hermelink
Talk: State and prove the theorem of the cube. Discuss the remark after the theorem and state and prove Corollary 1.
8. **The theorem of the cube: II, 17.12.2018**
Assigned reading: [3, §6, p. 56–59]
Speaker: C. Gürer
Talk: State and prove Corollaries 2, 3 and 4 and the Proposition on page 57. Discuss Application 1 and 2.
9. **Dividing varieties by finite groups I, 07.01.2018**
Assigned reading: [3, §7, p. 62–65]
Speaker: O. Hoyer
Talk: Recall the definition of étale morphism and prove the main theorem (on page 63) of §7.
10. **Dividing varieties by finite groups II, 14.01.2018**
Assigned reading: [3, §7, p. 66–70]
Speaker: J. Hermelink
Talk: Present the remaining results of §7, starting from page 66.
11. **The dual abelian variety: char 0, I, 21.01.2018**
Assigned reading: [3, §8, p. 70–73]
Speaker: T. Ose
Talk: Present the results of §8, ending with the proof of Theorem 1.
12. **The dual abelian variety: char 0, II, 28.01.2018**
Assigned reading: [3, §8, p. 74–78]
Speaker: C. Stadlmayr
Talk: Present the remaining results of §8, starting after the proof of Theorem 1.
13. **Cohomology of line bundles, 04.02.2018**
Assigned reading: [3, §16, p. 140–144]
Speaker: M. Paulsen
Talk: Prove the Riemann–Roch theorem and the vanishing theorem for abelian varieties.

Literatur

- [1] R. Hartshorne, *Algebraic Geometry*, Springer, New York, 1977.
- [2] J.S. Milne, *Abelian varieties*, <https://www.jmilne.org/math/CourseNotes/AV.pdf>
- [3] D. Mumford, *Abelian Varieties*, Tata Institute of Fundamental Research, Mumbai, Corrected Reprint 2014.