Aufgabe 5 16.11.2010

Let M a closed submanifold of a manifold M' with $\dim M < \dim M'$. Denote $H := TM'|_M$.

a) Show that H is a vector bundle over M, and that TM is a subbundle of H.

b) Show that there exists a natural vector bundle N over M with $TM \oplus N = H$. Of course, by general results we know that $TM \subset H$ has a complement in H (cf. section 21). But here we want a construction coming from the situation $M \subset M'$.

c) In the case of $M = S^2 \subset \mathbb{R}^3 = M'$ give 2 different complements N (defined by geometrical constructions).

d) Under which additional assumptions (or structures) on M' can we expect a unique complement N of TM in H?

e) Prove or disprove: If $E \oplus F$ and F are trivial for vector bundles E and F over M, then E is trivial as well.

<u>f) Main part of Blatt 5:</u> In the case of $M' = \mathbb{R}^4$ try to induce a connection on TM by projecting natural derivatives in $M' = \mathbb{R}^4$ to M (resp. TM). What about general M'?