

Aufgabe 4

9.11.10

Notiztitel

04.11.2010

Let $\pi_E : E \rightarrow M$, $\pi_F : F \rightarrow M$ be vector bundles over the manifold M .

a) Let $B \in T_2^0(E)$ be a bilinear tensor (given as $B \in T_2^0(\Gamma(M, E))$ or as a section in the corresponding vector bundle $E^* \otimes E^*$). Discuss the maps $E \rightarrow E^*$, $\Gamma(M, E) \rightarrow \Gamma(M, E^*)$ and $\Gamma(M, E) \rightarrow \Gamma(M, E)^*$ induced by B and check under which conditions they are isomorphisms. Converse? Is a morphism $E \rightarrow E^*$ induced by a B ? And an isomorphism?

b) Show that a suitable interpretation the pullback $\pi_F^* E$ is isomorphic to $E \otimes F \rightarrow M$.

c) Describe $E^* \otimes F$ & $\mathcal{L}(E, F)$ explicitly by bundle charts and establish the isomorphism $E^* \otimes F \rightarrow \mathcal{L}(E, F)$.