Aufgabe 12

18.1.11

1. Let M be a manifold M and let $R = R(TM)$ be the frame bundle of the tangent bundle. Prove that there exists a natural bijection between the set of connections on R (a connection forms) and the set connections on TM (as covariant derivatives).

2. Let (M, g) be a semi Riemannian manifold with its Levi-Civita-connection on TM. This connection defines a connection form on the orthonormal frame bundle $O(TM)$ (Proof!) Show that there is a bijection between the set of metric covariant derivatives (Levi-Civita conn.) and the set of connections on $O(TM)$.

3. What about the same question about a vector bundle E over M and $R(E)$?