Aufgabe 11

11.1.11

Mathematical Gauge Theory WS 10/11

1. Let R denote the frame bundle of a vector bundle E of rank r and let E be given by a cocycle (g_{ij}), $g_{ij} \in \mathcal{E}(U_{ij}, GL(r, \mathbb{K}))$. Give a detailed proof of: The pfb P defined by the same cocycle (g_{ij}) is isomorphic to R as a pfb.

2° Similarly, let (P, π, M, G) be a pfb given by a cocycle (g_{ij}), $g_{ij} \in \mathcal{E}(U_{ij}, G)$ and let $\varrho : G \to GL(r, \mathbb{K})$ be a representation. Prove: The associated vector bundle $E_\varrho = P \times_\varrho \mathbb{K}^r$ is isomorphic (as a vector bundle) to the vector bundle given by $\varrho(g_{ij})$. Details!

3° In the situation of a free action of a Lie group G on a mfd P such that the orbit space $M := P/G$ exists: Describe how to obtain a cocycle defining the corresponding pfb.

4° Discuss further problems or properties where the „cocycle description" of pfbs can be used, e.g. determining the morphisms between two pfbs or testing whether a given vector bundle is the associated bundle $P \times_\varrho \mathbb{K}^r$, ...