Aufgabe 10 21.12.10

Notiztitel Mathematical Gauge Theory WS 10/11

1. Show that the group $W_n \cong \mathbb{Z}/_{n\mathbb{Z}}$ of the $n^{th}$ roots of unity acts on $\mathbb{C}$ as a 0-dim. Lie group.

i) Does the orbit space $\mathbb{C}/_{W_n}$ exist?

ii) For the annulus $A = A(r, R) := \{z \in \mathbb{C} : r < |z| < R\}$, $0 \leq r < R \leq \infty$, does the orbit space $A/_{W_n}$ exist?

In case of existence: Determine $B \subset \mathbb{C}$ such that the orbit space and $B$ are conformally isomorphic.

2. On $M = S^1 \times S^1 \subset \mathbb{C} \times \mathbb{C}$ consider the $U(1)$ action

$$U(1) \times M \longrightarrow M, \quad \left(e^{it}, (z, w)\right) \longmapsto \left(e^{i\alpha t} z, e^{i\beta t} w\right).$$

i) Check that is a left (or right) action of the Lie group $U(1)$.

ii) Determine the condition on $\alpha, \beta$ under which the orbit space $M/_{U(1)}$ exists.

3. Assume that the Lie group $G$ acts freely on a mfd $M$ with the property: For all compact subsets $K \subset M$ the set $\{g \in G : K \cap gK \neq \emptyset\}$ is finite ($G$ "acts properly discontinuous"; „eigentlich diskontinuierlich"). Show that the orbit space $M/G$ exists by checking that the equivalence relation $R = R_G \subset M \times M$ is a closed submanifold.