THE GORDON-LEWIS PROPERTY IN VECTOR VALUED FUNCTION SPACES

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1. Introduction and Notations

A Banach space X has the Gordon-Lewis property if every absolutely summing operator defined on X factors through an L^1-space. This concept was introduced in connection with the solution of a conjecture posed by Grothendieck [GL]. This property is known to be shared by a number of important classes of Banach spaces such that $C(K)$ and $L^p(\mu)$ $(1 \leq p \leq \infty)$ or, more generally, Banach spaces whose biduals are isomorphic to complemented subspaces of Banach lattices. Several authors such as Cassaza, Nielsen or Reisner have analyzed different aspects of this property as well as some other related structures [CN1], [CN2], [R1], [R2]. Indeed, the vector-valued stability of this property has been studied in [R2], where it is proved that $\ell^p(X)$ $(1 \leq p < \infty)$ has the Gordon-Lewis property, whenever X has it (see also [CN1, Theorem 1.21], for another proof when $p = 2$). We mention that Reisner's result does not pass over to the case $p = \infty$, because ℓ^∞ does not have an unconditional basis.

With entirely different techniques, we complete these previous results. Namely, we prove that if X has the Gordon-Lewis property, then so do $L^p(\mu, X)$ $(1 \leq p \leq \infty)$ and $C(K, X)$, where μ is an arbitrary (not necessarily purely atomic) finite measure and K is an arbitrary compact Hausdorff space.

Throughout this paper (Ω, Σ, μ) denotes a probability space, X a Banach space and $L^p(\mu, X)$ (resp. $L^\infty(\mu, X)$) the space of (classes of) of Bochner integrable (resp. essentially bounded and measurable) functions defined over Ω. Likewise, $C(K, X)$ denotes the Banach spaces of all X- valued continuous functions defined on the compact Hausdorff set K.

In this paper, we are only concerned with two operator ideals: the p -summing operators and the 1-factorizable operators (see [DJT, Chapters 2 and 9]). The corresponding norms will be denoted by π_p and γ_1, respectively. So, a Banach space X has the Gordon-Lewis property if there is a constant c such that

$$\gamma_1(T) \leq c\pi_1(T),$$

for every 1-summing operator T from X to an arbitrary Banach space. We label the smallest of these constants as $gl(X)$.

In general, we refer the reader to [Di] for Banach space theory, to [DU] for the theory of vector measures, to [DJT] for the theory of absolutely summing operators and to the survey of [H] for the theory of ultrapowers in Banach spaces.

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2. Results

We state first two technical results which may have some interest in their own.

Proposition 2.1. *Let X and Y be Banach spaces, (A_n) be a pairwise disjoint sequence of Σ-measurable subsets and $T_n : X \to Y$ $(n \in \mathbb{N})$ be a sequence of 1-factorizable operators with* $\sup\{\gamma_1(T_n) : n \in \mathbb{N}\} < \infty$. *Then, the operator $V : L^1(\mu, X) \to Y$ defined as*

$$V(f) := \sum_{n=1}^{\infty} T_n \left(\int_{A_n} f d\mu \right) \in Y, \; f \in L^1(\mu, X),$$

is also 1-factorizable and $\gamma_1(V) \leq \sup\{\gamma_1(T_n) : n \in \mathbb{N}\}$.

Proof. It is clear that V is a well-defined operator with

$$\|V\| \leq \sup\{\|T_n\| : n \in \mathbb{N}\}.$$

Fix $\varepsilon > 0$. Since T_n is 1-factorizable, for each $n \in \mathbb{N}$ there exist a measure μ_n and two operators

$$S_n : X \to L^1(\mu_n), \quad R_n : L^1(\mu_n) \to Y,$$

such that $\|S_n\| \leq 1$, $\|R_n\| \leq \gamma_1(T_n) + \varepsilon$ and $T_n = R_n S_n$.

We denote by L the 1-sum of the spaces $L^1(\mu_n)$ $(n \in \mathbb{N})$, by J_n the canonical injection of $L^1(\mu_n)$ into L and, by P_n the canonical projection from L into $L^1(\mu_n)$. Define the following two operators ,

$$\begin{aligned} S &: L^1(\mu, X) \to L^1(\mu, L), \quad f \longmapsto \sum_{n=1}^{\infty} J_n S_n(f(\cdot))\chi_{A_n}(\cdot). \\ R &: L^1(\mu, L) \to Y, \quad f \mapsto \sum_{n=1}^{\infty} \int_{A_n} R_n P_n(f(\omega)) d\mu. \end{aligned}$$

Since the operators S_n are bounded, we see that S is a well-defined operator. Moreover, given $f \in L^1(\mu, X)$, we have

$$\|S(f)\| = \int \left\| \sum_{n=1}^{\infty} J_n S_n(f(\omega))\chi_{A_n}(\omega) \right\| d\mu \leq \sum_{n=1}^{\infty} \|S_n\| \int_{A_n} \|f(\omega)\| \, d\mu \leq \|f\|_1 .$$

That is, $\|S\| \leq 1$.

On the other hand, since the operators R_n are bounded, the mapping R is also well-defined. Moreover, given $f \in L^1(\mu, L)$, we have

$$\begin{aligned} \|R(f)\| &\leq \sum_{n=1}^{\infty} \left\| \int_{A_n} R_n P_n(f(\omega)) d\mu \right\| \leq \sum_{n=1}^{\infty} \|R_n\| \int_{A_n} \|f(\omega)\| \, d\mu \\ &\leq \sum_{n=1}^{\infty} (\gamma_1(T_n) + \varepsilon) \int_{A_n} \|f(\omega)\| \, d\mu \leq (\sup\{\gamma_1(T_n) : n \in \mathbb{N}\} + \varepsilon) \|f\|_1 . \end{aligned}$$

That is, $\|R\| \leq \sup\{\gamma_1(T_n) : n \in \mathbb{N}\} + \varepsilon$.

To show that V is 1-factorizable, it is enough to check that $V = RS$, since $L^1(\mu, L)$ is an L^1-space. So, take $f \in L^1(\mu, X)$ and compute.

$$\begin{aligned}
(RS)(f) &= R\left[\sum_{n=1}^{\infty} J_n S_n(f(\cdot))\chi_{A_n}(\cdot)\right] \\
&= \sum_{m=1}^{\infty} \int_{A_m} R_m P_m \left[\sum_{n=1}^{\infty} J_n S_n(f(\omega))\chi_{A_n}(\omega)\right] d\mu \\
&= \sum_{n=1}^{\infty} \int_{A_n} R_n P_n J_n S_n(f(\omega)) d\mu = \sum_{n=1}^{\infty} \int_{A_n} T_n(f(\omega)) d\mu \\
&= \sum_{n=1}^{\infty} T_n \left(\int_{A_n} f(\omega) d\mu\right) = V(f).
\end{aligned}$$

The above equality also implies that

$$\gamma_1(T) \leq \|R\| \, \|S\| \leq \sup\{\gamma_1(T_n) : n \in \mathbb{N}\} + \varepsilon.$$

Since ε was arbitrary, we finally deduce that

$$\gamma_1(T) \leq \sup\{\gamma_1(T_n) : n \in \mathbb{N}\}.$$

□

Lemma 2.2. *Let $1 \leq p \leq \infty$. Let X be a Banach space with the Gordon-Lewis property. Suppose $T : L_p(\mu, X) \longrightarrow Y$ is $1-$summing. Consider an infinite countable Σ^+ -partition $\pi = \{A_n : n \in \mathbb{N}\}$ and define the operators $T_n : X \longrightarrow Y$ $(n \in \mathbb{N})$ given by*

$$x \mapsto T_n(x) := T\left(\frac{1}{\mu(A_n)} x \chi_{A_n}\right).$$

Let $\phi_\pi := \sum_{n=1}^{\infty} \pi_1(T_n)\chi_{A_n}$. Then $\phi_\pi \in L^q(\mu)$ and ϕ_π allows the norm estimation

$$\|\phi_\pi\|_q \leq \pi_1(T),$$

where $q \in [1, \infty]$ and conjugate to p.

Proof. We have to prove that

$$\left\| \phi_\pi := \sum_{n=1}^{\infty} \pi_1(T_n)\chi_{A_n} \right\|_q \leq \pi_1(T).$$

We start with the case $q = \infty$. In this situation $T : L^1(\mu, X) \to Y$, so $T_n = T\Delta_n$, where

$$\Delta_n : X \to L^1(\mu, X), \quad x \mapsto \frac{1}{\mu(A_n)} x \chi_{A_n}.$$

Hence,

$$\pi_1(T_n) \leq \pi_1(T) \, \|\Delta_n\| \leq \pi_1(T)$$

and

$$\|\phi_\pi\|_\infty \leq \sup\{\pi_1(T_n) : n \in \mathbb{N}\} \leq \pi_1(T).$$

Now, let $1 \leq q < \infty$. Then, for some element (r_n) of the unit sphere of ℓ^p $(1 < p \leq \infty)$, we have

$$\|\phi_\pi\|_q = \left(\sum_{n=1}^{\infty} \pi_1(T_n)^q \mu(A_n) \right)^{1/q} = \sum_{n=1}^{\infty} \pi_1(T_n) r_n \mu(A_n)^{1/q}.$$

In what follows, we use the agreement $0 = 1/\infty$. We define, for $p \in (1, \infty]$,

$$f := \sum_{n=1}^{\infty} \frac{r_n}{\mu(A_n)^{1/p}} \chi_{A_n}.$$

Then,

$$\begin{aligned}
\|f\|_p^p &= \sum_{n=1}^{\infty} r_n^p \mu(A_n)^{-1} \mu(A_n) \leq 1 \quad (1 < p < \infty), \\
\|f\|_\infty &= \sup\{r_n : n \in \mathbb{N}\} \leq 1 \quad (p = \infty) \quad \text{and, for } 1 < p \leq \infty \\
\int_\Omega f \phi_\pi d\mu &= \sum_{n=1}^{\infty} r_n \mu(A_n)^{-1/p} \pi_1(T_n) \mu(A_n) = \sum_{n=1}^{\infty} \pi_1(T_n) r_n \mu(A_n)^{1/q} = \|\phi_\pi\|_q .
\end{aligned}$$

Fix $\varepsilon > 0$. Applying that every T_n is 1-summing, there exists a finite collection $x_1(n), \ldots, x_{m_n}(n) \in X$ $(n \in \mathbb{N})$ such that

$$\pi_1(T_n) \leq \sum_{j=1}^{m_n} \|T_n(x_j(n))\| + \varepsilon$$

and

$$(*) \quad \sup \left\{ \sum_{j=1}^{m_n} |\langle x_j(n), x^* \rangle| : \|x^*\| \leq 1, \; x^* \in X^* \right\} = 1.$$

Therefore, if we denote $s_n = r_n \mu(A_n)^{-1/p}$

$$\begin{aligned}
\|\phi_\pi\|_q &\leq \sum_{n=1}^{\infty} \left(\sum_{j=1}^{m_n} \|T_n(x_j(n))\| \right) r_n \mu(A_n)^{1/q} + \varepsilon \sum_{n=1}^{\infty} r_n \mu(A_n)^{1/q} \\
&\leq \sum_{n=1}^{\infty} \left(\sum_{j=1}^{m_n} \|T_n(x_j(n))\| \right) s_n \mu(A_n) + \varepsilon \, \|(r_n)\|_p \left(\sum_{n=1}^{\infty} \mu(A_n) \right)^{1/q} \\
&\leq \sum_{n=1}^{\infty} \sum_{j=1}^{m_n} \|T(s_n x_j(n) \chi_{A_n})\| + \varepsilon
\end{aligned}$$

Fix $N \in \mathbb{N}$. Applying now that T is 1-summing and that $L^q(\mu, X^*)$ is a norming subspace of $L^p(\mu, X)^*$ $(1 < p \leq \infty)$, we can find a function g in the closed unit ball of $L^q(\mu, X^*)$ such that

$$\begin{aligned}
\sum_{n=1}^{N} \sum_{j=1}^{m_n} \|T(s_n x_j(n) \chi_{A_n})\| &\leq \pi_1(T)(1+\varepsilon) \sum_{n=1}^{N} \sum_{j=1}^{m_n} |\langle s_n x_j(n) \chi_{A_n}, g \rangle| \\
&\leq \pi_1(T)(1+\varepsilon) \sum_{n=1}^{N} \sum_{j=1}^{m_n} \left| \left\langle x_j(n), \int_{A_n} f g d\mu \right\rangle \right|
\end{aligned}$$

Appealing to $(*)$, we can compute further. Denote

$$c_n = \begin{cases} \| \int_{A_n} fgd\mu\|^{-1}, & \text{if } \int_{A_n} fgd\mu \neq 0 \\ 0, & \text{if } \int_{A_n} fgd\mu = 0 \end{cases}, n \in \mathbb{N}.$$

Then,

$$\begin{aligned}\sum_{n=1}^{N}\sum_{j=1}^{m_n}\left|\left\langle x_j(n), \int_{A_n} fgd\mu\right\rangle\right| &\leq \sum_{n=1}^{N}\left\|\int_{A_n} fgd\mu\right\|\left(\sum_{j=1}^{m_n}\left|\left\langle x_j(n), c_n\int_{A_n} fgd\mu\right\rangle\right|\right)\\ &\overset{(*)}{\leq} \sum_{n=1}^{N}\left\|\int_{A_n} fgd\mu\right\| \leq \int_\Omega |f|\,\|g\|\,d\mu \leq \|f\|_p \leq 1\end{aligned}$$

Since N is arbitrary, we have

$$\|\phi_\pi\|_q \leq \pi_1(T)(1+\varepsilon)+\varepsilon.$$

Finally, since ε is also arbitrary, we obtain $\|\phi_\pi\|_q \leq \pi_1(T)$. □

We are ready to prove the main result.

Theorem 2.3. *Let $1 \leq p \leq \infty$. A Banach space X has the Gordon-Lewis property if and only if $L^p(\mu, X)$ has it.*

Proof. One implication is trivial, since X is isomorphic to a complemented subspace of $L^p(\mu, X)$. For the other implication, fix an operator T from $L^p(\mu, X)$ into a certain finite-dimensional Banach space Y, where $p \in [1, \infty]$. According to [R1, Proposition 3.3], we must find $C > 0$, depending only on $L^p(\mu, X)$, such that

$$\gamma_1(T) \leq C\pi_1(T).$$

Our first step will show that, for every countable (finite or infinite) Σ^+-partition π of Ω, $\gamma_1(T\mathbb{E}_\pi) \leq gl(X)\pi_1(T)$, where $\mathbb{E}_\pi : L^p(\mu, X) \to L^p(\mu, X)$ denotes, as usual, the conditional expectation operator related to π.

Without lose of generality, we fix an infinite countable Σ^+ -partition $\pi = \{A_n : n \in \mathbb{N}\}$ and consider the operators $T_n : X \longrightarrow Y$ $(n \in \mathbb{N})$ given by

$$x \mapsto T_n(x) := T\left(\frac{1}{\mu(A_n)}x\chi_{A_n}\right).$$

For the sake of clearness, we assume that $T_n \neq 0$, for every $n \in \mathbb{N}$. Define $\phi_\pi := \sum_{n=1}^\infty \pi_1(T_n)\chi_{A_n}$. We know by Lemma 2.2 that ϕ_π belongs to $L^q(\mu)$ and $\|\phi_\pi\|_q \leq \pi_1(T)$, where $q \in [1, \infty]$ is the conjugate exponent of p.

Next we consider the operator $M_\pi : L^p(\mu, X) \longrightarrow L^1(\mu, X)$ given by

$$f \longmapsto \phi_\pi \cdot \mathbb{E}_\pi(f).$$

(the point indicates a pointwise multiplication). Then, if $f \in L^p(\mu, X)$, we have

$$\|M_\pi(f)\| \leq \int_\Omega |\phi_\pi|\,\|\mathbb{E}_\pi(f)\|\,d\mu \leq \|\phi_\pi\|_q\|f\|_p.$$

That is, $\|M_\pi\|_1 \leq \|\phi_\pi\|_q$.

Since $\pi_1(T_n) \neq 0$ and $\|T_n\| \leq \pi_1(T_n)$ $(n \in \mathbb{N})$, the operator $T_\pi : L^1(\mu, X) \longrightarrow Y$ given by

$$f \longmapsto \sum_{n=1}^{\infty} \frac{1}{\pi_1(T_n)} T_n \left(\int_{A_n} f d\mu \right)$$

is well-defined and, moreover, $T\mathbb{E}_\pi = T_\pi M_\pi$. To check this equality, we take $f \in L^p(\mu, X)$ and compute.

$$\begin{aligned}
T\mathbb{E}_\pi(f) &= T\left(\sum_{n=1}^{\infty} \frac{1}{\mu(A_n)} (\int_{A_n} f d\mu) \chi_{A_n}\right) = \sum_{n=1}^{\infty} T_n (\int_{A_n} f d\mu) \\
&= \sum_{n=1}^{\infty} \frac{1}{\pi_1(T_n)} T_n \left(\int_{A_n} \pi_1(T_n) f d\mu \right) = T_\pi \left(\sum_{n=1}^{\infty} \pi_1(T_n) \frac{1}{\mu(A_n)} (\int_{A_n} f d\mu) \chi_{A_n} \right) \\
&= T_\pi(\phi_\pi \cdot \mathbb{E}_\pi(f)) = T_\pi M_\pi(f)
\end{aligned}$$

Therefore,

$$\gamma_1(T\mathbb{E}_\pi) \leq \|\phi_\pi\|_q \gamma_1(T_\pi)$$

Since X enjoys the Gordon-Lewis property,

$$\gamma_1(T_n) \leq gl(X)\pi_1(T_n), \qquad n \in \mathbb{N}.$$

Therefore, applying Proposition 2.1 to the operator T_π and using Lemma 2.2, we deduce

$$\begin{aligned}
\gamma_1(T\mathbb{E}_\pi) &\leq \|\phi_\pi\|_q \sup\left\{ \gamma_1\left(\frac{1}{\pi_1(T_n)} T_n \right) : n \in \mathbb{N} \right\} \\
&\leq \pi_1(T) \sup\left\{ \frac{1}{\pi_1(T_n)} gl(X) \pi_1(T_n) : n \in \mathbb{N} \right\} = gl(X)\pi_1(T)
\end{aligned}$$

We end the proof of this theorem by means of an ultrafilter argument. Let $\mathcal{U}$ be an ultrafilter dominating the canonical order filter $\mathcal{F}$ defined on the set $\mathcal{I}$ of all countable (finite or infinite) Σ^+-partitions of Ω.

Fix $\varepsilon > 0$ and take an arbitrary $\pi \in \mathcal{I}$. Then, by the first step, we can find a measure μ_π and two operators

$$S_\pi : L^p(\mu, X) \to L^1(\mu_\pi) \text{ and } R_\pi : L^1(\mu_\pi) \to Y$$

such that

$$\|S_\pi\| \leq 1, \|R_\pi\| \leq (1+\varepsilon) gl(X)\pi_1(T) \text{ and } T\mathbb{E}_\pi = R_\pi S_\pi.$$

At this point, we consider three operators related to the above ultrapower. The first operator is defined as

$$J : L^p(\mu, X) \to (L^p(\mu, X))_{\mathcal{U}}, \quad f \mapsto (\mathbb{E}_\pi(f))_{\mathcal{U}}$$

The second operator is the ultrapower operator $S := (S_\pi)_{\mathcal{U}}$ which goes from $(L^p(\mu, X))_{\mathcal{U}}$ into $\left(L^1(\mu_\pi)\right)_{\mathcal{U}}$ and the third operator is given by

$$R : \left(L^1(\mu_\pi)\right)_{\mathcal{U}} \to Y, \ (f_\pi)_{\mathcal{U}} \mapsto \lim_{\mathcal{U}} R_\pi(f_\pi) \in Y.$$

We note that J and $(S_\pi)_{\mathcal{U}}$ are well-defined, since $\|\mathbb{E}_\pi\| \leq 1$ and $\|S_\pi\| \leq 1$, for all $\pi \in \mathcal{I}$. This also implies that $\|J\| \leq 1$ and $\|S\| \leq 1$.

The operator R is also well-defined, since the family of operators (R_π) is norm bounded and Y is finite-dimensional. In fact,

$$\|R\| \leq \lim_{\mathcal{U}} \|R_\pi\| \leq gl(X)\pi_1(T)(1+\varepsilon).$$

We want to show that $T = SRJ$. The proof of this equality requires the following fact:

$$\lim_{\mathcal{U}} \|\mathbb{E}_\pi(f) - f\|_p = 0, \text{ for every } f \in L^p(\mu, X) \text{ and every } p \in [1, \infty].$$

Indeed, this limit is even zero under the order filter $\mathcal{F}$. For $p \in [1, \infty)$, this can be checked following the arguments given in [DU, Chapter III, Lemma 2.1]. In this case, the basic ingredients of the proof are that simple functions are dense in $L^p(\mu, X)$ and that conditional expectations are eventually constant on these simple functions. For $p = \infty$, it is also possible to follow the arguments given in [DU, Chapter III, Lemma 2.1], but this time we must bear in mind that countably valued μ-measurable functions are dense in $L^\infty(\mu, X)$ [DU, Chapter II, Corollary 1.3] and that conditional expectations related to infinite countable partitions are eventually constant on this type of functions.
Hence, if $f \in L^p(\mu, X)$

$$\begin{aligned} (RSJ)(f) &= (RS)\left[(\mathbb{E}_\pi(f))_{\mathcal{U}}\right] = \lim_{\mathcal{U}} R_\pi(S_\pi(\mathbb{E}_\pi(f))) \\ &= \lim_{\mathcal{U}} (T\mathbb{E}_\pi)(\mathbb{E}_\pi(f)) = \lim_{\mathcal{U}} T(\mathbb{E}_\pi(f)) = T(\lim_{\mathcal{U}} \mathbb{E}_\pi(f)) = T(f) \end{aligned}$$

Since $\left(L^1(\mu_i)\right)_{\mathcal{U}}$ is again a L^1 -space [H], we have a 1-factorization of T with

$$\gamma_1(T) \leq \|R\| \, \|S\| \, \|J\| \leq gl(X)\pi_1(T)(1+\varepsilon).$$

Therefore, we deduce that $L^p(\mu, X)$ has the Gordon-Lewis property and, since $\varepsilon > 0$ is arbitrary, that $gl(L^p(\mu, X)) = gl(X)$.

□

Reisner and Cassaza-Nielsen presented their results in the more general context of p-sums of Banach spaces. The following corollary shows that they can be deduced from our theorem.

Corollary 2.4. *Let (X_n) be a sequence of Banach spaces verifying that* $\sup_n gl(X_n) < \infty$, *then $(\bigoplus_n X_n)_p$ has the Gordon-Lewis property, for every $1 \leq p \leq \infty$.*

Proof. Take a 1-summing operator T from $(\bigoplus_n X_n)_1$ into a Banach space Y. Define T_m the restriction of T to the subspace of $(\bigoplus_n X_n)_1$ formed by those sequences vanishing all the components except perhaps the m-position. Clearly, every T_m is 1-factorizable with $\gamma_1(T_m) \leq gl(X_m)\pi_1(T)$.

It is well known that $T = \sum_{n=1}^\infty T_n$ in the operator norm. Therefore, a suitable application of Proposition 2.1 tells us that

$$\gamma_1(T) \leq \sup\{\gamma_1(T_m) : m \in \mathbb{N}\} \leq \sup\{gl(X_m) : m \in \mathbb{N}\}\pi_1(T).$$

That is, $(\bigoplus_n X_n)_1$ has the Gordon-Lewis property.

Fix $p \in [1, \infty]$. Since $(\bigoplus_n X_n)_p$ is a complemented subspace of $\ell^p((\bigoplus_n X_n)_1)$, a joint application of Theorem 2.2 and the first part of this proof yield that $(\bigoplus_n X_n)_p$ has the Gordon-Lewis property. □

The vector-valued stability of the Gordon-Lewis property in $C(K,X)$ will be established using Theorem 2.2. For the sake of completeness, we recall that $L^1_{w*}(\mu, X^*)$ denotes the Banach space (of equivalence classes) of w^*-measurable functions $f : \Omega \to X^*$ for which there is $g \in L^1(\mu)$ with $\|f(\omega)\| \le g(\omega)$, μ-almost everywhere. The norm is the infimum of those $\|g\|_1$, where f runs through the corresponding equivalence class and two w^* -measurable functions f_1, f_2 are identified if, for each $x \in X$, we have that $\langle x, f_1(\omega)\rangle = \langle x, f_1(\omega)\rangle$, μ -almost everywhere.

Corollary 2.5. *A Banach space X has the Gordon-Lewis property if and only if $C(K,X)$ has it.*

Proof. Let T be a 1-summing operator from $C(K,X)$ into a certain Banach space Y . Since T is weakly compact, so does the adjoint $T^* : Y^* \longrightarrow rcabv(K,X^*)$. As usual, $rcabv(K,X^*)$ denotes the Banach space of all regular countable additive Borel measures on K of bounded variation with values in X^*, endowed with the variation norm. By [BL], we can find a positive Radon measure λ on K such that $T^*(y^*)$ is λ-continuous, for all $y^* \in Y^*$. Thus, we can truely consider T^* as an operator from Y^* into $L^1_{w^*}(\lambda, X^*)$. We denote this operator by S and the canonical isometry from $L^1_{w^*}(\lambda, X^*)$ into $rcabv(K,X^*)$ by J. Taking again adjoints, we obtain that $T^{**} = J^*S^*$ is also 1-summing. It is well-known that J^* is indeed a 1-projection, so S^* is 1-summing. Since $L^\infty(\mu, X)$ embeds isometrically into $(L^1_{w^*}(\mu, X^*))^*$ in the canonical way, the restriction of S^* to that subspace Z of $(L^1_{w^*}(\mu, X^*))^*$ is 1-summing and according to Theorem 2.2 is 1-factorizable. Denote by Π the canonical embedding of $C(K,X)$ into its bidual. The composition $J^*\Pi$ clearly maps $C(K,X)$ into Z. Hence, we deduce that T is 1-factorizable. □

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