INTRODUCTION TO BOHMIAN MECHANICS
SUMMER TERM 2016

Exercise Sheet 8

Exercise 1: ***Projection Valued Measures – PVMs***

Recall the introduction of measurement operators via projectors onto wave functions associated with eigenstates of the apparatus.

a) Let A, B be subsets of $\mathbb{R}^n$. Show that the indicator functions of the sets ($\mathbb{1}_{A/B}(x) = 1$ for $x \in A/B$ and 0 otherwise) are orthogonal projectors $P_{A/B}$. Make explicit how they act on functions.

b) Now assume that $\mathbb{R}^n$ is spanned by disjoint sets A_i: $\mathbb{R}^n = \bigcup_{i=1}^{\infty} A_i$. Check that the family of orthogonal projectors P_{A_i} has properties which are expected of measures: Additivity, normalisation, positivity.

c) In the description of a measurement using projectors as in the lecture, convince yourself that the free choice of prefactors c_α leads to orthonormality of the system wave functions φ_α.

d) You showed that the statistical average of measurement outcomes is given by the expectation value of the associated operator. In this setting, compute the variance of those measurement outcomes and compare with the known formula from standard quantum mechanics.

Exercise 2: ***Positive-Operator Valued Measures – POVMs***

In the lectures we saw that experiments are better (more generally) described by POVMs instead of PVMs. As an example for the use of POVMs you discussed an experiment with noise of the measurement apparatus. Show that the resulting POVM is not a PVM.

Make sure you understand the formalism of POVMs and their usage since they are both mathematically and physically at the core of quantum measurement theory.

Exercise 3: ***Experiments***

Consider an experiment $\mathcal{E}$, where a given system with wave function ψ is acted upon with an apparatus. Using the different readouts of the apparatus, given the noise function of the apparatus p (which encodes the systematic errors of the device) and a set of numbers attached to the readouts, describe this physical process in generality and think about its meaning regarding the nature of measurements.

Exercise 4: ***What We Measure***

"But it is in principle quite false to base a theory solely on observable quantities. Since, in fact, it is the other way around. It is the theory which describes what we can observe."

This quote is from Einstein, talking to Heisenberg.

Write a short essay (short is key here) about what you think of this. Is Einstein wrong or right? Why? What does his being right or wrong mean for our understanding of given physical theories? Do a little research on topics like surreal trajectories or claims of the form "Bohmian trajectories contradict quantum mechanics" and analyse such claims regarding your knowledge of both standard and Bohmian quantum mechanics as well as Einstein's quote. You are of course completely free in your conclusions, form of argument and choice of sources.

You have until Friday, July 1st to hand it in (either personally or via mail: `schlenga@math.lmu.de`). This will not be marked but instead be used to find open questions which need to be adressed.