INTRODUCTION TO BOHMIAN MECHANICS
SUMMER TERM 2016

Exercise Sheet 7

Exercise 1: ***Conditional and Effective Wave Function***
Consider a universe with only two particles, X and Y.

a) Show that generally the conditional wave function for one of the particles does not fulfill an autonomous Schrödinger equation.
b) Explicitly check that the effective wave function of one of the particles does indeed solve an autonomous Schrödinger equation.

Exercise 2: ***Uniqueness of Quantum Equilibrium***
Repeat the proof of the uniqueness of the ψ^2-measure by assuming that the functional g only depends on ψ – not on any derivatives of ψ and not on the configuration q. Make all steps explicit. *(You can find the paper by Ward Struyve containing the proof on the course webpage.)*

Exercise 3: ***Typical Empirical Distribution***
Assume that the universe with wave function Ψ is split into a subsystem and its environment as we did in the lecture, and that the subsystem is described by an effective wave function ψ.

a) Recall what the effective wave function is, especially compared to the conditional wave function, and apply that to this case.
b) Now we're interested in describing an empirical distribution of particles. To this end, assume that the subsystem is further divided into N subsubsystems, all of which have the same effective wave function φ. Show that then the wave function of the subsystem composed of all the subsubsystems is given by

$$\psi(x_1, \ldots, x_N) = \Pi_{i=1}^{N} \varphi(x_i)\,.$$

c) With this, show that the probability of finding all the particles of the subsubsystems X_i in a volume element $\mathrm{d}x_i$ conditionalised to Y is given as:

$$\mathbb{P}^{\Psi}(X_1 \in \mathrm{d}x_1, \ldots, X_N \in \mathrm{d}x_N | Y) = \Pi_{i=1}^{N} \left|\varphi(x_i)\right|^2 \mathrm{d}x_i.$$

(This is the conditional measure as introduced in the lectures.)