INTRODUCTION TO BOHMIAN MECHANICS
SUMMER TERM 2016

Exercise Sheet 3

Exercise 1: *Typicality*
For a continuous function $f : \mathbb{R} \to \mathbb{C}$ with $\int_{\mathbb{R}} |f(x)|^2 \, \mathrm{d}x = \|f\|_2^2 < \infty$, show that for the Lebesgue measure λ of the set of points where the function is larger than a given $\varepsilon > 0$ the following holds:

$$\lambda(\{x \in \mathbb{R} : |f(x)| > \varepsilon\}) \leq \frac{C}{\varepsilon^2},$$

where C is a positive constant. Determine C.

Exercise 2: *Rademacher Functions and Independence*
The Rademacher functions $r_k : [0,1) \to \{0,1\}$ map $x \in [0,1)$ to the k-th digit of its binary representation.

a) Use the Rademacher functions to write $x \in [0,1)$ as a series of the form $\sum_k x_k 2^{-k}$.

b) With the notation $\mathbb{P}_f(a) = \lambda\big(\{f^{-1}(a)\}\big)$ and
$\mathbb{P}_{f_1,\ldots,f_n}(a_1,\ldots,a_n) = \lambda\big(\{f_1^{-1}(a_1) \cap \cdots \cap f_n^{-1}(a_n)\}\big)$, show that

$$\mathbb{P}_{r_{k_1},\ldots,r_{k_n}}(\delta_{k_1},\ldots,\delta_{k_n}) = \prod_{l=1}^{n} \mathbb{P}_{r_{k_l}}(\delta_{k_l}),$$

where $\delta_{k_l} \in \{0,1\}$ for $l = 1 \ldots n$.

c) Compute for $n \geq l \in \mathbb{N}$:

$$\lambda\left(\left\{x \in [0,1) : \sum_{k=1}^{n} r_k(x) = l\right\}\right)$$