INTRODUCTION TO BOHMIAN MECHANICS
SUMMER TERM 2016

Exercise Sheet 1

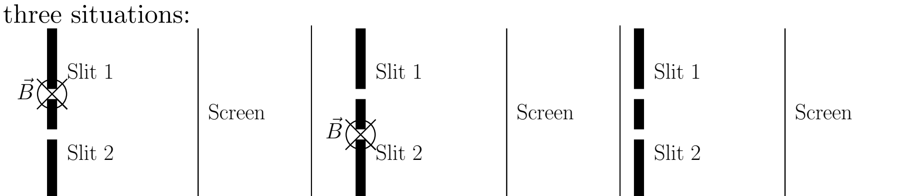
The exercise sessions take place on *Mondays, 14-16 in A027*. Please don't hesitate to ask questions (you can send them via mail to `schlenga@math.lmu.de` beforehand if you wish) and you're welcome to present and discuss solutions. Please visit the course webpage at
`http://www.mathematik.uni-muenchen.de/~schlenga/SS16/BohmMech.html`
from time to time to keep up with exercise sheets and general info about the class.
As a little warmup *homework*, you are invited to read Schrödinger's paper „Die gegenwärtige Situation der Quantenmechanik“ which will be posted to the webpage.

Exercise 1: ***The Wave Function***
This exercise deals with some properties of the wave function which are independent of specific physical setups.

a) Is it possible to design a measurement device – as it was presented in the lecture – to measure the wave function?
b) Remind yourself about the "cat problem", *i.e.* the state generated by coupling a macroscopic device (or a cat) to a system in a superposition and letting the coupled system evolve. Describe this process using density matrices and analyse the outcome for being a pure or a mixed state.

Exercise 2: ***The Double Slit***
The double slit experiment is claimed to reveal all important features of quantum mechanics. In this exercise, we investigate this experiment.

a) Consider first a classical double slit. A source shoots little electrically charged balls under random angles at a double slit and we accumulate their arrival points at a screen. Imagine three situations:



Slit 1 is filled with a magnetic field $\vec{B}$ which does not reach slit 2.	Now slit 2 is subjected to a magnetic field $\vec{B}$ and slit 1 is field-free.	Finally, both slits are without magnetic fields.

Qualitatively, how do you expect situation 3 to behave compared to situations 1 and 2? What does this have to do with quantum mechanics?

b) In quantum mechanics, the double slit experiment yields an interference pattern. You also know that "which-way"-information, *i.e.* measuring through which slit the particle moved, destroys this interference. Show this.
c) Usually in text-books, when calling the wave emanating from slit 1 $\psi_1(x,t)$ and that from slit 2 $\psi_2(x,t)$, the interference pattern on the screen is said to have the density shape $|\psi_1 + \psi_2|^2(x,t)$. Why is this not really correct?

Exercise 3: *Configuration Space*
The configuration space is not only important for quantum mechanics, where many of the paradoxical results can be explained by invoking the configuration space, but also in classical dynamical systems.

a) What is the configuration space M for N identical particles in $\mathbb{R}^3$? What special property do functions defined on M have?
b) In rural Bavaria, two roads connect villages A and B. Two cars, which are tied together with a rope of length $2L$, can both travel from A to B (along any of the roads) without tearing the rope. Is it possible for two large trucks with heavy cargo of radius $R > L$ to travel in opposite directions between villages A and B without running into each other?
   Hint: Draw a configuration space picture with coordinates x, y depicting the relative distance along the two roads.

Exercise 4: *Phase Space*

a) Show that solutions $x : \mathbb{R} \to \mathbb{R}$ of the differential equation $\dot{x} = x^2$ with initial condition $x(0) = x_0$ blow up in finite time.
b) For the one-dimensional physical pendulum with length L and pointlike mass m in Earth's gravity, draw and discuss the phase-space vector field generated by the dynamics. Include some integral curves. For small elongations, how can the integral curves be approximated?
c) Now draw the phase space vector field and integral curves for the one-dimensional harmonic oscillator of frequency ω. What is its flow Φ_t and does it fulfill the group property $\Phi_s \circ \Phi_t = \Phi_{s+t}$?
d) For the *two*-dimensional harmonic oscillator, explicitly compute div $v^H((q,p))$.