LMU Munich
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Exercises on Mathematical Statistical Physics II
Sheet 2

Problem 1 (Grönwall's Lemma) Let $g : \mathbb{R}_0^+ \to \mathbb{R}_0^+$ be integrable, $\varepsilon > 0$ and $f : \mathbb{R}_0^+ \to \mathbb{R}_0^+$ be continuous. Furthermore, let the left derivative of f be bounded by $g(t)(f(t) + \varepsilon)$, i.e.

$$\lim_{h \searrow 0} \frac{f(t) - f(t-h)}{h} - g(t)(f(t) + \varepsilon) \leq 0.$$

Show that for all $t \in \mathbb{R}_0^+$

$$f(t) \leq e^{\int g(s)ds}(f(0) + \varepsilon) \; .$$

Hint: Assume that there is a $t > 0$ and a $\eta > 0$ such that

$$f(t) \geq e^{\int g(s)ds + \eta t}(f(0) + \varepsilon + \eta) \; .$$

Show that this assumptions contradicts the assumptions above.

Problem 2 (Computation of "Term I") Let ψ be a solution of the many-body Schrödinger equation, φ a solution of the Hatree equation, p_j^φ and q_j^φ be defined as in exercise 1 on sheet 1. Let furthermore $\delta V_{12} := V(x_1 - x_2) - (V * |\varphi|^2)(x_1)$ be the difference between the "true" microscopic potential and the "effective" Hatree potential.

a) Compute all the commutators $[p_i^\varphi, q_j^\varphi] \; \forall i, j$.

b) Show explicitly that $|\langle \psi, \; p_1^\varphi p_2^\varphi \; \delta V_{12} \; q_1^\varphi p_2^\varphi \; \psi \rangle|$ vanishes identically. (Hint: Compute first $p_2^\varphi \; \delta V_{12} \; p_2^\varphi$)
Why is this fact so important?

Problem 3 (Computation of "Term IIIb") Under the conditions and definitions of exercise 2, find an upper bound for $|\langle \psi, \; q_1^\varphi p_2^\varphi \; \delta V_{12} \; q_1^\varphi q_2^\varphi \; \psi \rangle|$ with $\delta V_{12} \in L^\infty(\mathbb{R}^3, \mathbb{R})$.

The solutions to these exercises will be discussed on Friday, 11.11.2016.